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FOREWORD

The International Journal on Optimization and Applications (IJOA) is an open access, double blind peer-reviewed online journal aiming at publishing high-quality research in all areas of : Applied mathematics, Engineering science, Artificial intelligence, Numerical Methods, Embedded Systems, Electric, Electronic engineering, Telecommunication Engineering... the IJOA begins its publication from 2021. This journal is enriched by very important special manuscripts that deal with problems using the latest methods of optimization. It aims to develop new ideas and collaborations, to be aware of the latest search trends in the optimization techniques and their applications in the various fields..

Finally, I would like to thank all participants who have contributed to the achievement of this journal and in particular the authors who have greatly enriched it with their performing articles.

Prof. Dr. Hanaa HACHIMI Editor in Chief

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Unifying Multi-SIEM Ecosystems: A Risk-Aligned Approach to SOC Automation

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***Abstract*—In contemporary Security Operations Centers (SOCs), organizations often face the challenge of managing security incidents across multiple, heterogeneous client environments. Each client may rely on a distinct Security Information and Event Management (SIEM) solution, leading to fragmented visibility, inconsistent alerting formats, and disjointed incident handling procedures.**

This paper proposes a unified log management and automa tion framework designed to bridge the gap between these diverse SIEM environments. Deployed in a virtualized OpenStack environment, the solution is aligned with a manually constructed risk map inspired by EBIOS and ISO/IEC 27005, enabling rational prioritization of incident scenarios. The architecture introduces a centralized log management platform for normalized visibility and integrates orchestration mechanisms for automated, prioritized response.

This approach demonstrates that a lightweight, agile, and governance-aligned SOC is feasible even under constrained environments. This study details the technical implementation, the risk-based scenarios, the orchestration workflows, and the observed outcomes.

***Index Terms*—Security Operations Center, SIEM, SOAR, Wazuh, ELK Stack, Cortex XSOAR, Automated Incident Response, Open Source Tools, Risk Mapping, ISO 27005, EBIOS, OpenStack**

I. Introduction

In the era of escalating cyber threats, Security Operations Centers (SOCs) have become indispensable to organizational defense strategies. Their core mission is to ensure continuous monitoring, detection, and response to security events across enterprise networks. However, the operational complexity of modern SOCs has grown significantly due to the heterogeneity of security infrastructures, particularly in environments where multiple clients or business units rely on distinct Security Information and Event Management (SIEM) solutions.

This diversity often stems from legacy deployments, vendor preferences, compliance requirements, or budget constraints. While each SIEM platform offers specific capabilities, their coexistence within the same operational context leads to fragmented visibility, redundant alerts, and non-uniform inci- dent handling. Such fragmentation hinders SOC teams from detecting complex attack chains spanning multiple systems, correlating events efficiently, and executing timely, automated responses.

Traditional SIEMs are rarely sufficient to address the need for orchestration and large-scale automated response. They generally lack native mechanisms to prioritize alerts based on business impact, resulting in alert fatigue and inefficient resource allocation. Simultaneously, SOCs are increasingly expected to align their operations with governance, risk, and compliance (GRC) frameworks. Standards such as ISO/ IEC 27005 and methodologies like EBIOS require security operations to not only detect and respond to incidents but also to prioritize scenarios based on risk exposure and ensure traceable, structured decision-making.

To overcome these challenges, this paper introduces a unified log management and orchestration framework designed to integrate diverse SIEM environments with a risk-centric automation layer. The proposed architecture was deployed in a virtualized OpenStack environment, simulating the operational constraints of a multi-client SOC.

The key contributions of this work include:

- Centralized log management to normalize and unify events from multiple SIEM platforms.
- Automated incident response through an orchestration engine enriched with threat intelligence and ticketing mechanisms.
- Manual risk mapping, inspired by EBIOS and ISO/ IEC 27005, to rationally prioritize incident scenarios for automation.
- Lightweight architecture suitable for resource-con- strained SOCs, demonstrating the feasibility of achieving GRC alignment without sacrificing operational agility.

This work ultimately demonstrates the viability of a harmonized, risk-driven SOC capable of bridging technical silos and delivering effective responses under limited resources.

A. Paper Overview

This paper addresses the growing challenge faced by modern Security Operations Centers (SOCs) in managing fragmented SIEM environments across varied infrastructures. It introduces a unified architecture that harmonizes log collection, normalization, and automated response across diverse systems, with a strong alignment to organizational risk priorities. The study begins by contextualizing the limitations of current SOC operations, particularly in environments with multiple client infrastructures and disjointed log formats. A review of related research highlights the importance of orchestration and risk-based prioritization. The proposed solution is then deployed in a virtualized OpenStack testbed, integrating open-source tools including Wazuh, Filebeat, ELK stack, and Cortex XSOAR.

The methods detail the technical design of the platform, covering agent-based log ingestion, secure log forwarding, and enriched alert handling. Evaluation is conducted using quantitative metrics such as event volume, detection coverage, and system resource usage, complemented by a semi-quantitative risk analysis inspired by ISO 27005 and EBIOS RM.

Results demonstrate the viability of a lightweight, automated SOC architecture capable of operating under con- strained environments while maintaining high visibility, detection granularity, and risk-driven responsiveness. The paper concludes with a discussion on operational performance, risk impact reduction, and perspectives for scaling the solution.

B. Related Work

Security Information and Event Management (SIEM) systems have become foundational tools in modern Security Operations Centers (SOCs) for aggregating, correlating, and analyzing security events [1]. However, the proliferation of diverse SIEM solutions across organizations and clients intro- duces significant challenges related to data fragmentation, lack of interoperability, and inconsistent incident handling [2]. Researchers and practitioners have recognized the need for log centralization and normalization techniques to overcome these obstacles. Centralized log management platforms, often based on scalable architectures, provide a unified view of security data, enabling improved correlation and situational awareness [3].

Simultaneously, Security Orchestration, Automation, and Response (SOAR) platforms have emerged to address the op- erational inefficiencies in SOCs caused by high alert volumes and manual response workflows [4]. SOAR solutions enable automation of incident triage, enrichment, and remediation actions, improving response times and reducing analyst fatigue [2]. Yet, their effectiveness is contingent on consistent and enriched event data, underscoring the importance of seamless integration with SIEMs and log management systems [3].

Several studies have investigated approaches to integrate multiple heterogeneous SIEM systems within a unified secu- rity architecture. These works highlight challenges such as inconsistent data schemas, diverse communication protocols, and varying alert semantics, which complicate automation efforts [5]. To address these, layered architectures combining log normalization, centralized storage, and flexible orches- tration engines have been proposed [6]. However, most of these approaches focus either on improving interoperability or on orchestration, but do not demonstrate scalable, modu- lar, and resource-efficient deployments under realistic SOC constraints.

Furthermore, integrating risk management frameworks such as EBIOS and ISO/IEC 27005 into SOC workflows has gained attention. Risk-driven orchestration prioritizes incident handling based on business impact and threat likelihood, ensuring efficient allocation of resources and alignment with governance objectives [7]. Although the application of such frameworks in operational SOCs remains limited, their poten- tial to enhance SOC effectiveness and strategic alignment is well documented [8].

Compared to these works, our framework provides a uni- fied, modular architecture capable of handling diverse SIEM data, integrating risk-driven orchestration, and maintaining lightweight performance in a virtualized environment. This A combination of scalability, modularity, and governance alignment represents the key differentiator of our approach.

II. Methods

To validate the effectiveness of the proposed SOC framework, we implemented a modular architecture combining centralized log collection, automated analysis, and risk-centric orchestration. The deployment was carried out in a virtualized environment using OpenStack, simulating the operational constraints of a multi-tenant SOC managing heterogeneous infrastructures.

A. Testbed Environment Setup

The proposed architecture was deployed on four virtual ma- chines (VMs) within an OpenStack private cloud, along with a separate FortiGate firewall acting as an external log source. The environment was designed to emulate a realistic SOC infrastructure operating under resource constraints.

The virtual machines were provisioned with the following specifications:

ID de l'instance	Rôle	vCPUs	RAM	Disque	Système d'exploitation
ELK	Stack ELK (Kibana, Elasticsearch)	8	24 Go	20 Go	Ubuntu Server
Wazuh	SIEM Wazuh (manager + agent)	4	16 Go	300 Go	Ubuntu Server
Windows Server	Source de logs Windows	4	8 Go	200 Go	Windows Server
XSOAR	SOAR Cortex XSOAR	8	16 Go	200 Go	Ubuntu Server

Fig. 1: Configuration of the deployed virtual machines.

In addition to the virtual machines, a FortiGate firewall (outside of OpenStack) was configured to forward syslog- based security events to the Wazuh manager. Logs from both internal systems and the firewall were collected using Wazuh agents and Filebeat, then normalized and analyzed in the central SIEM pipeline.

B. Centralized Log Management Layer

Logs from all sources — including FortiGate and internal servers — are collected and normalized via:

- Filebeat and Wazuh agents, configured on VM 4 and on the FortiGate device, to ship logs to the central platform.
- VM 3 (192.168.50.183) hosts the Wazuh manager, which:
  - Ingests and parses raw logs,
  - Applies detection rules,
  - Generates structured alerts.json outputs.
- VM 2 (192.168.50.237) runs Elasticsearch for scalable storage and indexing.
- VM 1 (192.168.50.242) runs Kibana, offering dash- boards for visualization and analysis.

This design ensures cross-device log unification and facilitates correlation of heterogeneous events.

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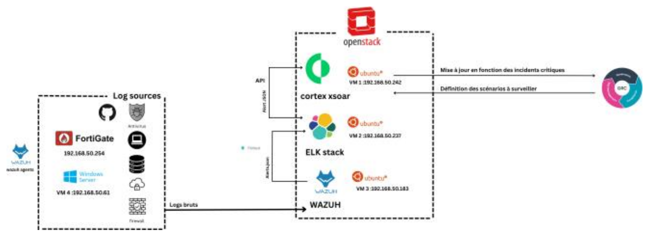

Fig. 2: Architecture du SOC léger déployé.

C. *Orchestration and Automation with XSOAR*

The automation component of the SOC was implemented using Cortex XSOAR, hosted on a dedicated VM within the OpenStack environment. This platform served as the orches- tration engine to execute predefined response playbooks. Key features include:

- Automated ingestion of structured alerts from Wazuh (via JSON files or API).
- Playbooks designed in XSOAR to automate triage, en- richment (e.g., with VirusTotal), and incident escalation.
- Conditional workflows, allowing decisions based on severity, source, or type of attack.
- Ticket creation or external system notifications (e.g., sending alerts to GRC or ITSM tools).

This layer enabled scalable, repeatable, and risk-aware inci- dent response, reducing analyst workload and standardizing SOC reactions to known scenarios.

D. *Risk Mapping Methodology*

To align the automation logic with organizational priorities, a manual risk mapping exercise was conducted based on:

- The EBIOS RM methodology and ISO/IEC 27005 risk analysis principles.
- Identification of key threat scenarios (e.g., lateral move- ment, credential theft).
- Prioritization based on likelihood and impact, influ- encing which alerts would trigger automated XSOAR playbooks.
- Scenarios were then translated into automation logic inside XSOAR, ensuring business relevance in response execution.

This mapping ensures SOC actions are aligned with gover- nance and business risk priorities.

E. *GRC Integration and Feedback Loop*

Critical incidents identified by Wazuh (on VM 3) are shared with an external GRC interface through an API-based con- nector:

- High-priority alerts trigger updates to risk registers and compliance actions,
- The GRC layer provides feedback on mitigation, guiding corrective security measures,
- This two-way exchange bridges technical operations and strategic risk governance.

F. *Evaluation Metrics*

The performance of the framework was evaluated using the following criteria:

- Log unification effectiveness across disparate formats (FortiGate syslog, Windows Event Logs),
- Detection latency and response automation,
- Playbook coverage, i.e., the proportion of high-risk sce- narios automated.
- Dashboard usability and analyst workflow in Kibana,
- Success rate of GRC integration actions,
- System performance under limited resource allocation.

III. Results

A. *Log Ingestion and Normalization*

Over a two-month observation period, the deployed SOC solution ingested and normalized more than 1.3 million events from heterogeneous sources, including FortiGate firewalls, Windows Servers, and Linux systems. Wazuh agents collected logs from each endpoint and forwarded them to the Wazuh manager, which parsed them into structured alerts. These alerts were then processed by Filebeat and sent securely to Elasticsearch. The use of custom Wazuh templates ensured consistent field mapping in Kibana.

The Kibana dashboard provided real-time visibility across all log sources. As shown in Figure 3, the most active hosts included 1-win- **and window-**, with log counts exceeding 200,000 records. The chart in Figure 4 shows the top ten triggered detection rules, highlighting that “Logon Failure – Unknown user or bad password” was the most frequent, with over 563,000 alerts.

Other dominant detections included registry changes and suspicious process creations, confirming both breadth and depth of detection capabilities. Agent distribution, shown in Figure 3 and 4, demonstrates widespread endpoint coverage, reinforcing the system’s ability to detect both misconfigura- tions and attack indicators.

This level of visibility confirms the effectiveness of the log ingestion and normalization pipeline, supporting robust monitoring and downstream automation workflows.

The visualizations below provide supporting evidence of the log ingestion and normalization process. They confirm both the diversity of log sources and the effectiveness of the central processing pipeline.

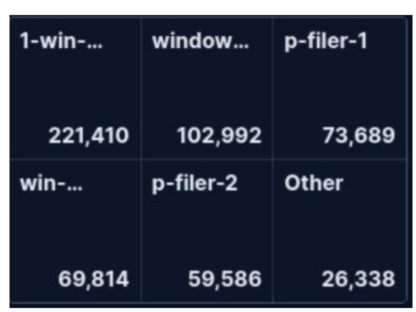

Fig. 3: Distribution of logs per host.

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Fig. 4: Top 5 values of agent IPs reporting logs.

B. Incident Detection and Playbook Automation

Following the successful normalization of log data from multiple sources, the automated response layer was activated through Cortex XSOAR. Alerts generated by Wazuh were fil- tered based on severity and rule identifiers, then automatically ingested into XSOAR using a dedicated API connector. Each ingested alert triggered a predefined playbook, designed to execute a set of actions depending on the nature of the threat. These playbooks typically included contextual enrichment (e.g., IP reputation checks, geolocation), classification (inter- nal vs. external threat), and escalation mechanisms (case creation, email notification, or risk flagging). For instance, repetitive failed login attempts detected by Wazuh were au- tomatically classified as brute-force attempts, enriched with WHOIS and VirusTotal lookups, and escalated for analyst validation.

The automation workflow also ensured that alerts mapped to critical business scenarios—identified during the risk map- ping phase—were prioritized. Playbooks were aligned with these scenarios to allow immediate triage and reduce analyst intervention in routine detections. This orchestration reduced average response time, improved consistency in incident han- dling, and ensured that high-risk alerts followed a traceable and standardized path.

The figures below illustrate how alert categories and their volume influenced the triggering of playbooks and automated responses.

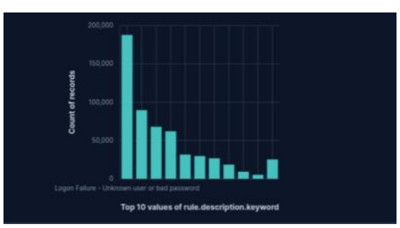

Fig. 5: Most frequent rule categories mapped to automated response work- flows.

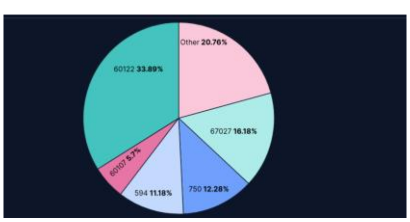

Fig. 6: Pie chart of rule volume distribution

C. Risk Scenario Coverage

To ensure that incident response workflows were not only au- tomated but also aligned with business impact, a manual risk mapping process was conducted prior to the deployment of XSOAR playbooks. This mapping was inspired by the EBIOS Risk Manager methodology and the ISO/IEC 27005 standard. It involved identifying critical risk scenarios based on realistic threats observed across the ingested logs (e.g., brute-force attacks, privilege escalation, persistence techniques).

Each scenario was assessed according to two main factors: likelihood (based on frequency and attack patterns in the logs) and impact (based on the asset or business process involved). The resulting scenarios were ranked and used to guide the design of XSOAR playbooks.

Inside XSOAR, alerts matching these risk scenarios were tagged and classified accordingly. For example:

- Brute-force login attempts were mapped to a scenario labeled "Unauthorized access to critical systems."
- Registry changes and persistence mechanisms were linked to "Malware persistence on sensitive endpoints."
- Suspicious command execution or lateral movement signs were associated with "Internal propagation of compromise."

These tags influenced the playbook logic, enabling tailored actions such as enrichment depth, escalation level, or direct GRC notification. The mapping ensured that automation remained risk-aware, focusing response efforts on the most business-critical threats while deprioritizing low-risk, routine noise.

This strategic classification not only improved incident prioritization but also created a transparent link between detection events and organizational risk posture, enhancing the traceability of SOC decisions and aligning operational activity with governance expectations.

D. System Performance and Resource Usage

To evaluate the operational efficiency of the deployed SOC infrastructure, we monitored system resource usage over sev- eral days. The environment, hosted on an OpenStack-based virtual infrastructure, included the Wazuh manager, Elastic- search cluster, Kibana dashboard, and XSOAR orchestrator.

As illustrated in Figure 7, the platform demonstrated con- sistent stability over a continuous uptime exceeding three days. The system's total memory allocation was 23.5 GB, with approximately 19.3 GB utilized during peak operation. Despite intensive log ingestion and indexing tasks, swap memory remained minimally used, confirming the effectiveness of resource allocation and tuning.

CPU distribution across eight logical cores shows moder- ate-to-high utilization, particularly on cores handling Elastic- search and Kibana processes. Elastic agents and indexers (visible in the process list) operated under acceptable load, without performance degradation.

Overall, the architecture fulfilled its design goal of offer- ing a lightweight yet capable SOC solution. Performance remained stable even while handling over 1.3 million alerts, supporting real-time visualizations, rule matching, and automation workflows.

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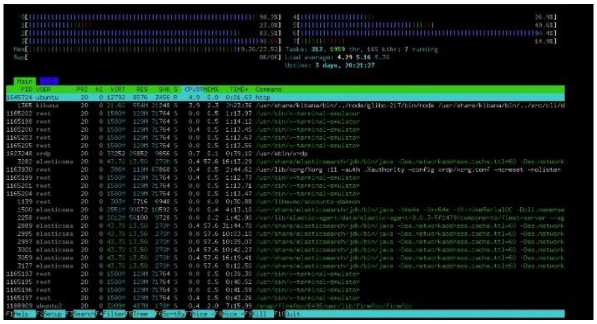

Fig. 7: System resource usage (htop view after 3 days of operation).

E. Risk Prioritization and Reduction

To assess the effectiveness of the proposed SOC automation framework from a governance perspective, a semi-quantitative risk evaluation methodology was employed. This approach aligns with ISO/IEC 27005 and EBIOS RM standards, com- bining threat probability and business impact into a criticality score used to prioritize mitigation efforts.

Evaluation Methodology Risk was calculated using the standard formula:

`Risk=Probability×Impact`

Two axes were used:

- Probability (1 to 4): from rare to frequent
- Impact (1 to 4): from minimal to critical consequences These were then combined into a 4x4 risk matrix, allowing a visual classification of threats from low to critical. The evaluation scales used are shown in Figure 8. **Results and Interpretation**

Each identified scenario was assigned a risk score before and after the deployment of the automated SOC system. The graph in Figure 9 shows the evolution of risk levels (R1 to R6), with a notable reduction in criticality thanks to proactive detection, prioritization, and response orchestration.

Before mitigation, R1 and R2 initially exhibited the highest criticality due to their significant impact or higher frequency. After mitigation, their levels dropped to minimal values, illus- trating the effectiveness of the measures applied.

R4 to R6 experienced a substantial decrease as well, transitioning from critical or elevated levels to acceptable post-mitigation values.

This progression highlights how the implemented controls successfully reduced the overall risk exposure, demonstrating the alignment between detection workflows and governance- driven prioritization.

Figures

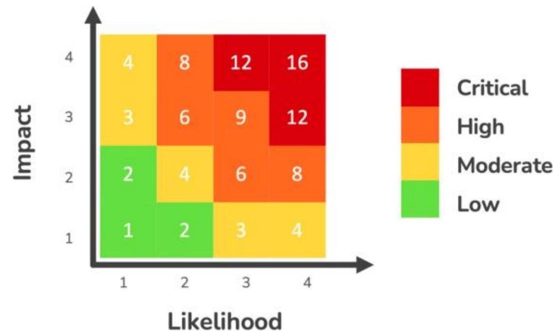

Fig. 8: Risk Matrix Based on Impact and Probability).

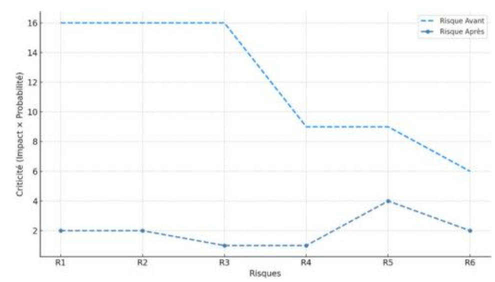

Fig. 9: Evolution of Risk Levels (Before vs. After Mitigation).

IV. Conclusion and Perspectives

In an increasingly fragmented cybersecurity landscape, orga- nizations often struggle to unify incident detection and response across varied client environments. This study ad- dressed that challenge by proposing a harmonized SOC archi- tecture capable of bridging diverse SIEM ecosystems through centralized log management and risk-aligned automation.

The solution, deployed in a virtualized environment, inte- grated multiple security tools—including Wazuh, Filebeat, the ELK stack, and Cortex XSOAR—while maintaining a light- weight and modular footprint. Over a two-month evaluation period, the system ingested and normalized over 1.3 million logs, achieving high visibility across endpoints, networks, and firewalls. Automated workflows enabled enriched alert processing and consistent playbook-based responses.

A complementary risk evaluation framework, inspired by ISO/IEC 27005 and EBIOS RM, confirmed that the approach improved both operational performance and the overall risk posture. The transition from manual to orchestrated responses reduced the criticality of multiple scenarios, particularly brute-force attempts and misconfiguration-related threats.

Despite these promising results, the project faced con- straints: the testbed does not fully replicate production-scale environments, and orchestration scenarios were limited in scope.

Future work will focus on four directions:

Benchmarking the framework against enterprise SOC plat- forms such as Splunk SOAR, IBM QRadar, and Microsoft Sentinel to evaluate scalability, response time, and resource efficiency.

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Integration of AI-driven anomaly detection to enhance dynamic behavior analysis.

Extension of orchestration playbooks to cover insider threats, cloud-based attacks, and lateral movement scenarios.

Deployment of hybrid-cloud architectures and real-time risk scoring feedback loops to improve scalability and strengthen GRC alignment.

Ultimately, this project demonstrates that harmonizing multi-SIEM ecosystems is both technically feasible and oper- ationally valuable. It lays the foundation for building SOCs that are lightweight, risk-driven, and proactively aligned with governance and compliance objectives.

V. Appendix A – Integration details and Configuration Highlights

This appendix describes the technical configuration and integration steps implemented during the SOC deployment, focusing on secure data flow, normalization, and orchestration mechanisms.

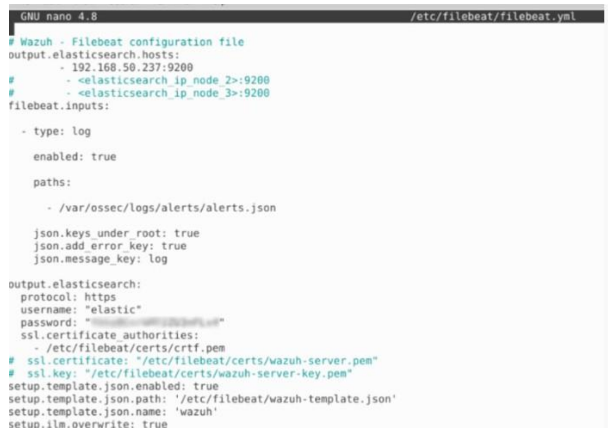
A. Installation and Configuration of Wazuh Agents

To collect telemetry from endpoints and internal servers, Wazuh agents were installed on all relevant machines, includ- ing the Windows Server (VM 4) and Linux-based log sources. These agents were configured to monitor system activity such as authentication logs, file integrity events, and process execution. All agents were connected to the Wazuh manager hosted on VM 3 via secure channels using default ports 1514 and 1515. The configuration enforced mutual authentication and allowed for real-time forwarding of logs from distributed endpoints to the central Wazuh instance, which served as the detection layer of the SOC.

B. Integration of Wazuh with the ELK Stack via Filebeat

The integration between Wazuh and the ELK stack was carried out using Filebeat, which was installed on the same host as the Wazuh manager (VM 3). Filebeat was configured to read the JSON-formatted alerts generated by Wazuh from the path /var/ossec/logs/alerts/alerts.json. These alerts were parsed and forwarded to Elasticsearch (VM 2) using a secure HTTPS connection. TLS encryption was enabled through the use of SSL certificates and keys stored locally in /etc/filebeat/ certs/. Authentication was enforced using a dedicated Elastic- search user account. In addition, a Wazuh-specific template was loaded to ensure that all ingested data was properly struc- tured and indexed, allowing seamless visualization in Kibana (VM 1).

The screenshot below presents the actual configuration file used (/etc/filebeat/filebeat.yml), demonstrating how Filebeat was securely linked to Elasticsearch while applying Wazuh templates for optimized field mapping:



```
GNU nano 4.8                                   /etc/filebeat/filebeat.yml
# Wazuh - Filebeat configuration file
output.elasticsearch.hosts:
        - 192.168.50.237:9200
#         - <elasticsearch_ip_node_2>:9200
#         - <elasticsearch_ip_node_3>:9200
filebeat.inputs:

  - type: log

    enabled: true

    paths:

      - /var/ossec/logs/alerts/alerts.json

    json.keys_under_root: true
    json.add_error_key: true
    json.message_key: log

output.elasticsearch:
  protocol: https
  username: "elastic"
  password: "[illegible]"
  ssl.certificate_authorities:
    - /etc/filebeat/certs/crtf.pem
#  ssl.certificate: "/etc/filebeat/certs/wazuh-server.pem"
#  ssl.key: "/etc/filebeat/certs/wazuh-server-key.pem"
setup.template.json.enabled: true
setup.template.json.path: '/etc/filebeat/wazuh-template.json'
setup.template.json.name: 'wazuh'
setup.ilm.overwrite: true
```

Fig. 10: Filebeat configuration for Wazuh–ELK integration with TLS encryp- tion and Wazuh templates.

This setup ensured that all alerts from Wazuh were transmit- ted securely and interpreted correctly by Elasticsearch and Kibana, supporting centralized analysis and dashboarding.

C. Filebeat-to-XSOAR Integration via API

To enable automated incident response, a connector was developed to forward critical Wazuh alerts to Cortex XSOAR through its RESTful API. A Python script was implemented to read the JSON alerts generated by Wazuh, filter high-priority events, and convert them into the required format for XSOAR. These transformed alerts were then sent to the XSOAR / incident/add endpoint using an HTTP POST request. Authen- tication was handled using an API token associated with a restricted XSOAR user. The script was executed periodically via a scheduled cron job, ensuring near real-time ingestion of new incidents. This mechanism enabled automated playbook execution within XSOAR, with actions such as enrichment, classification, and escalation being triggered based on prede- fined logic. The integration was designed to be scalable and modular, making it possible to adapt the playbooks to different threat scenarios as defined in the risk mapping process.

D. Security Considerations and Best Practices

Throughout the architecture, secure communication was prior- itized to maintain data integrity and confidentiality. All con- nections between Filebeat and Elasticsearch were encrypted using TLS certificates, while access to the Elasticsearch instance was restricted to authenticated users. API interactions with XSOAR were secured using HTTPS and token-based authentication, and sensitive credentials were stored in pro- tected configuration files. Additionally, strict firewall rules were implemented to limit access to critical services such as Elasticsearch, XSOAR, and the Wazuh manager. Input validation mechanisms were applied during alert parsing and transformation to prevent injection risks or malformed data entries. These measures collectively ensured that the SOC infrastructure adhered to strong cybersecurity hygiene and reduced the attack surface of its operational components.

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Linking Management Control Systems to Performance Evaluation in Public Sector Enterprises: A Configurational Approach Based on Effectiveness, Efficiency, and Quality

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***Abstract* -In the context of ongoing reforms in the public sector, the evaluation and management of performance have become critical challenges. This article investigates the contribution of management control systems (MCS) to the improvement of organizational performance, considering its multidimensional nature, efficiency, effectiveness, and quality. Using a configurational approach (fsQCA) complemented by a weighted mathematical modeling, the study reveals multiple contextual and organizational configurations that explain overall performance. Findings highlight the central role of organizational culture, budgetary simplification, and information system modernization, while demonstrating an equifinality logic. The confrontation between fsQCA results and quantitative simulation illustrates the complementarity of both approaches, offering a richer understanding of the complex interplay between managerial practices and performance outcomes. Theoretically, this research advances the literature by combining configurational and weighted approaches. From a managerial perspective, it provides a framework for performance evaluation and steering, tailored to the specificities of public enterprises in Morocco.**

Keywords: Public sector; Performance Evaluation; Management Control Systems Configurational Approach; Weighted Modeling.

I. INTRODUCTION

In recent decades, public enterprises have been at the center of profound structural and managerial transformations, largely driven by reforms inspired by New Public Management (NPM). These transformations aim to enhance public sector performance by strengthening managerial accountability, introducing results-oriented approaches, and promoting efficiency in resource management. Within this context, the management control system (MCS) has progressively become an essential tool for steering performance.

However, while the adoption of management instruments borrowed from the private sector has modernized certain processes, it has also raised significant concerns. The specificities of the public sector, multiplicity of objectives, institutional constraints, political pressure, and the need for legitimacy vis-à-vis diverse stakeholders, render a simple transposition of private management models inadequate (Anthony, 1965; Ferreira & Otley, 2009). In the public sector, the MCS goes beyond its traditional role of budgetary oversight to act as a mechanism of mediation, regulation, and legitimation between often contradictory demands: regulatory compliance, performance expectations, social equity, and economic efficiency.

Accordingly, public performance cannot be understood in a one-dimensional way. It requires an integrated perspective encompassing three fundamental dimensions:

Effectiveness, referring to the ability to achieve the objectives set, in line with the mission of public service (Pollitt & Bouckaert, 2011; Boyne, 2002).

Efficiency, emphasizing the rational use of resources in relation to the outcomes achieved (OECD, 2005; Dunleavy et al., 2006).

Quality, which encompasses users' perceived quality, service standards, and continuous improvement mechanisms (Behn, 2003; Osborne & Gaebler, 1992).

Nevertheless, several studies stress that the influence of MCS on these performance dimensions is neither linear nor isolated. Its effects result from complex interactions among multiple contextual factors, including organizational culture, leadership style, accountability structures, and the level of digitalization of information systems (Malmi & Brown, 2008; Gerdin, 2005). In this regard, conventional analytical approaches struggle to fully capture such complexity.

To address this, the present study employs a configurational approach, and more specifically the fuzzy-set Qualitative Comparative Analysis (fsQCA) method. This approach enables the identification of multiple causal configurations, that is, combinations of conditions that jointly lead to high performance. It also recognizes the principle of equifinality: multiple, distinct pathways may result in the same favorable outcome, depending on the specific interplay of factors.

Drawing on empirical research conducted with a sample of Moroccan state-owned enterprises with a commercial orientation, this study pursues three main objectives:

To empirically assess the impact of MCS on public performance across its effectiveness, efficiency, and quality dimensions.

To identify the combinations of organizational and institutional factors that, in interaction with MCS, lead to high performance.

To discuss the theoretical and practical implications for the design and adaptation of control systems in public sector contexts.

By adopting both an analytical and contextual perspective, this study complements previous findings centered on global performance configurations. It provides a more refined understanding of the specific levers of performance across its different dimensions, thereby offering concrete insights for public decision-makers while enriching the academic debate on contemporary public management.

II. Literature reviews

1. From State Modernization to Public Enterprise Reform: A Global Dynamic

Since the 1980s, the movement of state reform, largely driven by the doctrines of New Public Management (NPM), has profoundly transformed the functioning of public administrations and organizations on a global scale (Hood, 1991; Dunleavy & Hood, 1994). This managerial paradigm, initially developed in Anglo-Saxon countries, was built on the desire to break away from the bureaucratic rigidities of the traditional Weberian model, in favor of management approaches focused on results, process efficiency, and a culture of performance. The introduction of private-sector-inspired mechanisms, such as contractualization, management by objectives, managerial accountability, and performance evaluation, represents one of the major hallmarks of this evolution.

Within this context, public enterprises, long regarded as state-owned entities dedicated primarily to public service missions, have been directly affected by these transformations. They are now expected to adopt more efficient managerial practices while preserving their commitment to the public interest (Christensen & Lægreid, 2011; Pollitt & Bouckaert, 2017). This hybridization of public and private logics raises significant challenges in terms of steering, regulation, and performance, particularly in institutional environments characterized by political, cultural, and administrative constraints.

In Morocco, this dynamic has been reflected in a series of ambitious reforms, embedded within a broader vision of modernizing the public sector. The reform of state-owned firms has been placed at the core of government priorities, with the aim of rationalizing resources, strengthening transparency, and improving both the economic and social performance of these entities. Initiatives such as the introduction of performance contracts, the redesign of evaluation mechanisms, the enhancement of governance frameworks, and the implementation of performance indicators illustrate this progressive transformation of the commercial public sector. These reforms are also aligned with a wider framework of public managerial accountability, consistent with international recommendations regarding good governance and fiscal discipline.

Thus, the reform of public firms in Morocco is fully embedded within the global trajectory of state modernization, while seeking to adapt the principles of NPM to the country's specific structural, institutional, and socio-economic realities.

2. The Management Control System in Public Organizations: Between Adaptation and Hybridization

The management control system (MCS) has become an essential tool for steering performance within public organizations. Borrowed from private-sector practices, it relies on a combination of tools and mechanisms such as dashboards, reporting, internal audits, budgetary control, and performance evaluation (Otley, 1999; Simons, 1995). These mechanisms enable public managers to plan, coordinate, monitor, and adjust actions in a way that aligns strategically with the objectives set by political authorities or stakeholders.

However, the application of standard management control models in public organizations is not straightforward. Their effectiveness largely depends on the degree to which they are adapted to the institutional, cultural, and political specificities of each context (Ferreira & Otley, 2009). In environments characterized by strong administrative traditions, such as in many Global South countries, the MCS often operates within a logic of hybridization. This results in the coexistence of private-sector-inspired instruments, bureaucratic practices inherited from classical administration, and symbolic uses aimed at ensuring compliance or legitimacy (Hassenteufel, 2020).

In Morocco, this hybridization is manifested in the tension between modernization demands, driven by recent reforms seeking to enhance accountability and performance, and constraints related to administrative culture, centralized decision-making, and limited managerial autonomy. MCS tools are sometimes implemented superficially, without genuine appropriation by actors, limiting their transformative potential. In this context, the MCS cannot be reduced to a set of neutral technical instruments: it becomes a socially constructed system, shaped by multiple logics, political, organizational, and cultural, that must be analyzed contextually.

3. Public Performance: A Plural, Multidimensional, and Politically Constructed Concept

The notion of performance in the public sector fundamentally differs from that in the private sector, where financial profitability is a central criterion. In public organizations, performance is multidimensional and cannot easily be reduced to standardized quantitative indicators. It encompasses complementary dimensions such as effectiveness (the ability to achieve set objectives), efficiency (the relationship between resources mobilized and results achieved), service quality, equity in access to public services, user satisfaction, and the sustainability of outcomes produced (Van Dooren et al., 2015; Talbot, 2010).

This plurality makes the evaluation of public performance particularly challenging. It requires measurement tools capable of capturing not only immediate results but also delayed, qualitative, and sometimes subjective effects. Moreover, the very definition of performance criteria is the subject of negotiation among various actors, political decision-makers, managers, citizens, and social partners, who may pursue divergent interests. From this perspective, public performance emerges as a political and social construct rather than a simple objective measure (Moynihan, 2008).

Consequently, public performance evaluation systems are often criticized for their excessive technicality or normative approach, which tends to standardize realities that are inherently diverse. To address these limitations, several authors advocate for a more integrated evaluation, based on a weighted approach that accounts for the institutional context, strategic priorities, and the relative importance assigned to each performance dimension (Bouckaert & Halligan, 2008). Such an approach would not only enhance the relevance of evaluation mechanisms but also strengthen their legitimacy among internal and external stakeholders.

4. Towards a Configurational Logic: Moving Beyond Linear and Contingency Approaches

Research on management control systems (MCS) has long been dominated by contingency approaches, which assume an «optimal» fit between certain contextual characteristics, such as organizational size, strategy, or environment, and the control mechanisms adopted (Chenhall, 2003; Donaldson, 2001). While these approaches have provided useful insights, they rely on a linear and isolated view of causal relationships, where each variable exerts a direct and unambiguous effect on performance. In public sector environments, characterized by high institutional complexity and sometimes conflicting objectives, this perspective proves insufficient for understanding the realities of performance management.

In response to the limitations of these deterministic models, configurational approaches offer a richer theoretical and methodological alternative. Rather than seeking a single solution or « best practice », they aim to identify specific combinations of contextual, structural, and managerial conditions that collectively explain performance (Fiss, 2011). This perspective is based on three fundamental principles: the complementarity of factors, equifinality (i.e., the existence of multiple pathways leading to the same outcome), and causal asymmetry, meaning that a condition present in one pathway to performance may be absent in another.

Within this framework, the fuzzy-set Qualitative Comparative Analysis (fsQCA) method emerges as particularly well-suited. By combining logical rigor with contextual sensitivity, it allows for the modeling of complex causal configurations in the form of recipe leading to a given outcome, in this case the performance of public organizations (Ragin, 2008; Misangyi et al., 2017). It also enables the identification of combinations of factors that are sufficient or necessary for achieving a result, considering interdependence, substitution, or mutual reinforcement among conditions.

In the institutional contexts of emerging countries such as Morocco, where reform dynamics are often characterized by hybridization, uncertainty, and the coexistence of multiple logics (administrative, managerial, political), this approach allows researchers to move beyond classical explanatory frameworks. It provides a more nuanced and realistic understanding of the factors that interactively contribute to enhancing performance in public enterprises.

5. Complementarity between Configurational Logic and Weighted Performance Modeling: An Original Methodological Contribution

An underexplored area in the scientific literature concerns the integration of configurational approaches, such as fsQCA, with weighted performance modeling tools. Although often applied separately, these two analytical frameworks offer a fruitful methodological complementarity. On the one hand, configurational analysis allows for the identification of causal profiles, or recipes, linking various combinations of contextual and managerial conditions to organizational performance. On the other hand, weighted modeling, typically based on multi-criteria techniques, provides a synthetic and integrated quantification of overall performance, considering its multiple dimensions (effectiveness, efficiency, quality, satisfaction, etc.).

Few studies combine these two approaches, even though they can significantly reinforce each other (Greckhamer et al., 2018; Bell et al., 2020). fsQCA provides a detailed, qualitative, and conditional understanding of explanatory factors, capturing interaction effects and equifinality. In turn, weighting performance criteria allows for the construction of an aggregated score, facilitating comparisons across cases and providing a robust measure useful for public decision-makers. This mixed approach thus overcomes the methodological compartmentalization still present in research by integrating qualitative and quantitative dimensions within a cross-validation logic.

Practically, this complementarity enables:

The identification of causal configurations leading to performance via the fsQCA method.

The estimation of a weighted performance score based on multidimensional criteria, reflecting the priorities of the studied context.

The comparison of profiles of high-performing and low-performing organizations, enhancing the robustness of results through triangulation.

This analytical coupling constitutes an original methodological contribution, particularly suited to complex contexts such as Moroccan public enterprises, where performance challenges are multidimensional and highly contextualized. It also provides public decision-makers with tools that are both explanatory and operational.

III. Theoretical and Conceptual Framework

1. *Rethinking Public Sector Performance: Towards a Multidimensional Approach*

In a context of public administration transformation, marked by the spread of New Public Management (NPM) principles, the notion of performance has gradually become a structuring reference for public organizations (Bouckaert & Halligan, 2008; Pollitt & Bouckaert, 2011). Unlike in the private sector, public performance cannot be reduced to financial or profitability indicators. It follows a multidimensional logic, aiming to reconcile sometimes conflicting objectives: meeting social expectations, optimizing resources, and ensuring sustainable service quality for citizens.

From this perspective, three essential dimensions emerge for the evaluation of public performance: effectiveness, efficiency, and quality.

Effectiveness refers to the capacity of a public organization to achieve the strategic and societal objectives assigned to it (Boyne, 2002). It measures the alignment between pursued goals (mission, public mandate) and achieved results, often within a framework characterized by multiple imperatives.

Efficiency concerns the relationship between resources mobilized and results produced. Within a logic of budgetary accountability, it becomes a central criterion for legitimizing public policies, particularly through productivity indicators, unit costs, or timelines (Dunleavy et al., 2006; OECD, 2005).

Quality constitutes a transversal and subjective dimension, encompassing not only compliance with public service standards but also user satisfaction, service responsiveness, accessibility, and personalization. It is often assessed using frameworks such as the Common Assessment Framework (CAF) or the EFQM model (Osborne & Gaebler, 1992; Behn, 2003).

These dimensions are interdependent and should be integrated within a logic of overall performance, where management becomes an exercise in balancing institutional constraints, societal expectations, and available resources.

2. *Management Control Systems: Steering Tools and Performance Levers*

The management control system (MCS) constitutes a strategic device for decision-making and performance management, structuring processes of evaluation, planning, measurement, and organizational learning (Simons, 1995; Otley, 1999). In public organizations, the MCS serves a triple function: ensuring internal regulation, reporting to external stakeholders, and supporting the continuous improvement of services (Van Dooren, Bouckaert & Halligan, 2010).

Historically inspired by private-sector practices, the MCS in the public sector has evolved into a hybrid model, considering the institutional, political, and cultural specificities of public organizations (Anthony, 1965; Ferreira & Otley, 2009). Unlike normative models, the public MCS is not a standardized or universally applicable tool. Rather, it becomes an instrument of mediation between bureaucratic requirements (compliance, legality, administrative control) and emerging performance expectations (results, public value, operational efficiency).

In this sense, the MCS also acts as a lever of legitimation, enabling public managers to justify their decisions, demonstrate responsiveness, and produce a « trace » of performance in environments often characterized by high political or social pressure.

3. *The MCS in the Public Sector: Between Contextual Constraints and Organizational Configurations*

Recent studies emphasize that the MCS does not exert a direct and isolated effect on performance (Malmi & Brown, 2008; Gerdin, 2005). On the contrary, its impact depends heavily on interactions with other contextual factors, such as:

Organizational structure and the degree of centralization.
Organizational culture and dominant values.
Leadership style and team mobilization capacity.
Adopted strategy and political orientations.
Institutional, legal, and regulatory environment.

These complex interactions suggest the need for a paradigm shift: moving from a linear logic to a configurational logic. In other words, the goal is no longer to identify a « best practice » or an « optimal » MCS model, but rather to understand which combinations of conditions produce performance effects in each context.

This perspective aligns with both contingency and configurational approaches, which posit that there is no single path to performance, but multiple « performance pathways » (Miller, 1981; Meyer et al., 1993; Fiss, 2007). Each configuration represents a specific arrangement of factors that, taken together, can lead to equivalent outcomes, reflecting the principle of equifinality.

4. *The fsQCA Approach: Modeling Complex Causal Relationships in the Public Sector*

To account for this complexity, the fuzzy-set Qualitative Comparative Analysis (fsQCA) method, developed by Ragin (2000, 2008), offers an appropriate methodological response. This approach enables the identification of causal configurations explaining high (or low) performance levels while simultaneously incorporating multiple conditions.

The strength of fsQCA lies in its ability to:

Account for multiple pathways to the same outcome (equifinality).

Model non-linear and asymmetric interactions between variables.

Reflect the complex and contextual realities of the public sector.

Integrate subjectivity (through fuzzy membership scores) while maintaining comparative rigor.

Within the scope of this article, the use of the configurational approach through fsQCA thus aims to better understand how management control systems, in interaction with other contextual factors, can contribute to the overall performance of public enterprises, measured across the dimensions of effectiveness, efficiency, and quality.

IV. Methodology

1. Epistemological Positioning and Methodological Choice

This research adopts an interpretivist stance, viewing management control and performance evaluation practices as socially situated constructs shaped by institutional, cultural, and organizational dynamics. It employs an abductive logic, facilitating iterative exchanges between empirical observations and theoretical frameworks to uncover explanatory configurations grounded in field realities (Dubois & Gadde, 2002).

The choice of the fsQCA (fuzzy-set Qualitative Comparative Analysis) method, developed by Ragin (2000, 2008), is justified by the aim to explore complex, non-linear causal relationships between organizational practices and overall performance. Unlike traditional statistical approaches, fsQCA enables the identification of combinations of conditions (or causal configurations) that may lead to the same performance level according to the principle of equifinality. It is particularly suitable for studies with a medium number of cases in heterogeneous institutional contexts.

2. Field, Sample, and Data Collection

The study was conducted on a theoretical sample of 25 Moroccan public enterprises with a commercial character, selected from the 57 establishments targeted by the performance reform, representing a sampling rate of 43.8%, well above the 20% threshold often considered the minimum in comparative case studies (Yin, 2018). These enterprises operate in various sectors (transportation, energy, infrastructure, etc.) and are particularly exposed to modernization and performance demands.

Data were collected using a methodological triangulation combining:

Questionnaires administered to managers responsible for control, planning, and evaluation.

Semi-structured interviews with senior executives.

Documentary analysis of activity reports, performance audits, budget documents, and internal indicators.

This plurality of sources allows cross-checking managerial discourse against formal practices, thereby enhancing the internal validity of the data.

3. Modeling Overall Performance

The dependent variable, « overall performance », was modeled based on three fundamental dimensions derived from public management literature:

Effectiveness: achievement of strategic objectives defined by the state.

Efficiency: optimization of financial, human, and technical resource use.

Service quality: level of satisfaction perceived by users or beneficiaries.

A multi-criteria weighting of these dimensions was performed based on expert consensus and empirical validation to create an aggregated overall performance score. This score serves both as the dependent variable in the fsQCA analysis and as a benchmark for numerical comparisons (cross-simulation).

4. Selection and Calibration of Causal Conditions

Based on an extensive literature review and a preliminary exploratory survey, six explanatory conditions were selected for their potential to interact with overall performance:

Table 1 : Causal Conditions: Synthetic Definitions

Condition	Synthetic Definition
Organizational Culture	Degree of shared values, norms, and meaning among members of the organization
Tenure in Position	Length of time managers have held their positions
Hierarchical Levels	Pyramid or flat structure of decision-making levels
Budget Simplification	Degree of fluidity in budgeting processes
IT Modernization	Integration of management and reporting technologies
Cooperation	Degree of collaboration between national and international partners

Source: self autors

These conditions were calibrated as fuzzy-set scores ranging from 0 to 1, following Ragin's methodology (2008), with three thresholds defined: non-membership (0), crossover/ambiguous point (0.5), and full membership (1). Thresholds were established using descriptive statistics and empirically validated against case data.

Table 2 : Causal condition calibration

Condition	0 (Non-membership)	0.5 (Crossover)	1 (Full membership)
Organizational Culture	< 0.25	0.50	> 0.75
Cooperation	< 0.30	0.60	> 0.80
Budget Simplification	< 0.20	0.50	> 0.80
IT Modernization	< 0.20	0.40	> 0.70
Tenure in Position	< 2 years	4 years	> 6 years
Hierarchy (levels)	> 6 levels	4 levels	< 2 levels

Source: self autors

5. fsQCA Analysis

The analysis was performed using fsQCA 4.1, following these steps:

Setting a minimum frequency threshold of 1 for inclusion in the analysis.

Setting the consistency threshold at 0.80 to ensure the relevance of retained configurations.

Extracting sufficient causal configurations, distinguishing intermediate and parsimonious solutions.

- Constructing a truth table based on observed combinations of conditions.

Tableau 3 : Extract from the truth table

Configuration	Culture	Cooperation	SPB	MSI	Tenure	Hierarchy	Consistency	Performance
1	1	1	1	1	1	0	0.95	Effectiveness
2	1	0	1	1	1	1	0.91	Efficiency
3	1	1	0	1	1	1	0.87	Quality
…	…	…	…	…	…	…	…	…

Source: self autors

The results identified several combinations of conditions leading to high performance levels, illustrating the equifinality logic inherent in the configurational method. These results were then cross validated with the weighted overall performance scores to test the coherence and complementarity between qualitative insights and quantitative validation.

6. Proposed Research Model

Based on the literature review and empirical results, public enterprise performance cannot be understood in a linear or isolated manner but rather through a combination of interdependent factors. fsQCA analyses revealed that overall performance arises from specific configurations incorporating organizational culture, IT modernization, budget simplification, inter-organizational cooperation, as well as governance-related factors and managerial tenure.

These results confirm the configurational and equifinal logic (Fiss, 2011; Misangyi et al., 2017), whereby multiple pathways can lead to the same high-performance outcome. They also align with studies highlighting management control systems as structuring mechanisms of public performance (Ferreira & Otley, 2009; Bouckaert & Halligan, 2018).

Concurrently, constructing a weighted performance evaluation model, based on a mathematical equation integrating coefficients for the different dimensions (effectiveness, efficiency, and quality), complements the configurational approach by providing a quantitative and comparative view of results. This methodological articulation between qualitative-comparative logic (fsQCA) and weighted quantitative simulation offers an original framework for analyzing public enterprises, linking strategic diagnosis and operational management.

Thus, the final research model (Figure 1) illustrates the interplay between:

Contextual and organizational conditions (culture, IT systems, cooperation, governance, etc.).

The management control system as a structuring mechanism.

Dimensions of overall performance (effectiveness, efficiency, quality). (Fig 1)

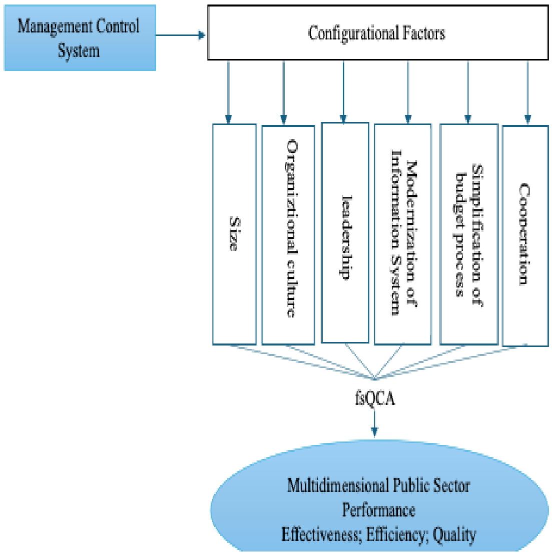

Fig.1: Integrated configurational Model linking management control system and multidimensional performance in public sector companies

V. RESULTS

1. Quantitative Evaluation of Public Enterprise Performance: Toward an Integrative Model

Evaluating the performance of public enterprises is a complex exercise due to their multiple missions, institutional constraints, and non-profit objectives. Observing the lack of robust tools specifically adapted to the Moroccan context, we developed a mathematical model for evaluating overall performance, complementing the qualitative and configurational analyses derived from the fsQCA method.

This model formalizes performance as a weighted function of three fundamental dimensions, Quality (Q), Effectiveness (E), and Efficiency (F), identified as central through our empirical analysis and validated by the study of high-performing causal configurations. The general equation proposed is as follows:

$$Perfi = Qi.Pq + Ei.Pe + Fi.Pf \quad \text{(eq.1)}$$

where Q, E, and F represent the scores obtained in the three dimensions, and Pq,Pe,Pf are the weighting coefficients reflecting their strategic importance according to the evaluated scenario.

To test the robustness and flexibility of the model, a series of numerical simulations was performed, varying the initial weights in the discrete set {0;0.5;1} and then extending them to the interval [0;1] in increments of 0.1. This variation allows modulating the impact of each dimension on overall performance and adapting the evaluation to the strategic priorities of each public enterprise or sector (e.g., prioritizing quality in healthcare or efficiency under tight budget constraints).

Simulations were conducted on a sample of Moroccan public enterprises, initially processed in Excel and subsequently analyzed using Python to visualize performance in three dimensions and detect dynamic patterns. The operational formula applied to each entity i is:(Eq1)

For example, a scenario with the following weighted scores:

Q·Pq=0.969, E·Pe=0.877, F·Pf=0.929 results in a total performance of 2.775. Using the average values observed in our sample (Q = 0.969; F = 0.929; E = 0.877) with equal weights provides a reference performance score, adjustable according to strategic priorities.

Based on this formalization, we developed an integrated performance evaluation table including:

Indicators grouped by dimension (Q, E, F) with a 10-point scale.

Weighted aggregation producing a global score.

Visual representation enabling quick identification of underperformance or optimal performance areas.

This table aims to serve as an operational management tool for public decision-makers, allowing objective evaluation, prioritization of action levers, and monitoring of performance evolution over time. This mathematical approach complements the qualitative fsQCA analysis by providing structured quantification of performance, exploring alternative scenarios via weight variation, facilitating inter-enterprise and inter-sector comparisons, and fostering a culture of evaluation and accountability in the Moroccan public sector.

2. Cross-Analysis: Causal Configurations and Quantitative Performance Modeling

A Dual Perspective on Organizational Performance

Our research results are based on two complementary approaches:

On one hand, a configurational analysis (fsQCA) identifying combinations of causal factors associated with high performance.

On the other hand, a weighted mathematical model providing direct quantification of performance across three fundamental dimensions: quality, effectiveness, and efficiency.

Each approach addresses a different aspect of the problem:

The fsQCA method highlights equifinality and the diversity of paths leading to performance.

The quantitative model, conversely, allows measurement of obtained performance, simulation of alternative scenarios, and decision-support tools.

3 Convergences: Centrality of Key Factors in Both Approaches

Table 4 : Cross-analysis reveals notable convergences:

Key Element	fsQCA Results	Performance Model
Organizational Culture	Present in 100% of high-performance configurations	Associated with the quality dimension
Simplification of Procedures	Frequently present in high-performance configurations	Contributes to efficiency through streamlined processes
IT Modernization	Important conditional variable in multiple solutions	Impacts both efficiency and quality
Internal/External Cooperation	Key factor in high-performance configurations	Enhances service quality and collective effectiveness
Tenure in Position	Present in some optimal combinations	Less directly quantifiable, but linked to organizational stability

Source: self autors

Thus, certain qualitative levers necessary for performance translate into measurable improvements in at least one of the Q, E, or F dimensions in the quantitative model.

4 Divergences and Complementarities

Some insights emerge only from one approach, highlighting their complementarity:

fsQCA reveals synergies among factors, e.g., a strong culture combined with internal cooperation and a modern IT system can achieve high performance even if efficiency is moderate.

The quantitative model allows:

Numerical estimation of overall performance.

Identification of imbalances between dimensions.

Customization of weights according to strategic priorities.

A public enterprise can exhibit high performance in the mathematical model (e.g., 2.7/3) without appearing in a high-performance fsQCA configuration if it lacks a key factor (e.g., cooperation) not captured in the quantitative equation.

5.Toward an Integrated Approach

We propose articulating both approaches in a hybrid logic. fsQCA identifies causal configurations underlying performance, guiding organizational reforms, management practices, and structural decisions. The weighted model serves as an operational monitoring tool, enabling performance measurement, scenario simulation, and dashboard construction for decision-makers. This combination allows identifying an optimal configuration, quantifying actual performance, and highlighting levers to sustainably improve future outcomes.

5.6 Methodological Contribution of Combining fsQCA and Weighted Modeling

This articulation provides a novel analytical framework for Moroccan public enterprises by combining:

Configurational abduction: to explore recurring patterns and explain diverse performance trajectories.

Quantitative deduction: to simulate, test, and evaluate the impacts of concrete actions.

This dual approach links strategic diagnosis (fsQCA) with operational performance management (weighted model), offering a comprehensive and enriched reading of public sector performance.

VI. Discussion: fsQCA and Public Performance

This section discusses fsQCA results in relation to classical public performance dimensions (effectiveness, efficiency, and quality) and theoretical frameworks. The cross-analysis provides insights into causal configurations in Moroccan public enterprises and situates them empirically in contemporary public management debates.

1. Organizational Culture as a Structuring Core

One major finding is the recurrence of organizational culture in all high-performance configurations, making it a necessary condition in our model. This aligns with Schein (2010), for whom culture is the invisible cement of organizational behavior, and Denison (1990), linking dimensions like involvement, consistency, and adaptability to overall performance. In Moroccan public enterprises, shaped by bureaucratic heritage and strong institutionalization, culture stabilizes and mediates between performance logic and ingrained practices.

This finding also echoes Bouckaert & Halligan (2008), who argue that public performance cannot rely solely on technical instruments without shared values.

2. Specific Combinations for Performance Dimensions

Configurational analysis identified multiple paths to performance, mobilizing different levers according to targeted dimensions:

Effectiveness: achieved through clarity of responsibilities, managerial stability, and alignment of objectives and resources (Boyne, 2002).

Efficiency: associated with modernized IT systems, streamlined procedures, and standardized processes, reflecting rationalization logic (OECD, 2005; Osborne & Gaebler, 1992; Dunleavy et al., 2006).

Service Quality: strongly linked to transversal cooperation, active user engagement, and organizational learning (Behn, 2003).

3. Equifinality and Systemic Complementarity

fsQCA demonstrates equifinality: multiple condition combinations can produce the same performance level. Examples include:

A configuration with strong cooperation, IT modernization, and short tenure achieves performance through collective dynamics and agility.

Another with long tenure, strong culture, and simplified procedures shows stability can produce positive effects if supported by appropriate tools.

These compensations confirm Malmi & Brown (2008) and Gerdin (2005): «It is not the MCS alone that influences performance, but its interaction with structural, strategic, and cultural dimensions». The management control system mediates tensions between bureaucratic requirements, result orientation, and political expectations.

4. Beyond Classical MCS Approaches in the Public Sector

Traditional MCS approaches inspired by the private sector (Anthony, 1965) focus on planning and control. In this context, MCS appears as:

A regulatory tool (Simons, 1995), managing tensions between innovation and compliance.

A legitimizing lever, structuring transparency and meeting oversight expectations.

A coordination space, integrating human resources, digital tools, and organizational rules.

This aligns with Van Dooren, Bouckaert & Halligan (2010), framing public performance as a cyclical process of production, evaluation, and learning. This study demonstrates that this process can only be modeled configurationally, sensitive to complex factor interactions.

VII. Conclusion, Contributions, Limitations, and Perspectives

This research aimed to explore how the management control system (MCS), approached from a configurational perspective, influences the different dimensions of performance in Moroccan public enterprises. Using a methodology based on fuzzy-set Qualitative Comparative Analysis (fsQCA), we analyzed combinations of causal conditions likely to explain high levels of organizational performance, assessed across its three fundamental dimensions: effectiveness, efficiency, and quality.

The results reveal multiple configurations, reflecting a logic of equifinality, in which organizational culture consistently plays a central role. This finding, recurrent across all high-performing combinations, highlights that public performance does not result solely from technical tools or formal control mechanisms but rests primarily on a structuring cultural foundation. These results corroborate the contributions of Denison (1990), Schein (2010), and Bouckaert and Halligan (2008), who emphasize that any public modernization effort relies on stakeholder engagement and the consolidation of shared organizational values.

Furthermore, the study highlighted specific pathways toward each performance dimension:

Effectiveness is fostered by clarity of responsibilities and strategic alignment.

Efficiency largely depends on the modernization of tools and the streamlining of procedures.

Quality relies on interdepartmental cooperation, user orientation, and organizational learning.

This cross-analysis demonstrates that public performance cannot be conceived monolithically. It results from complex interactions among structural factors (procedures, information systems), human factors (experience, cooperation), and cultural factors. In this regard, the fsQCA approach proves particularly fruitful for overcoming the limitations of linear approaches and for highlighting logics of complementarity, substitution, and mutual reinforcement among factors.

Contributions of the study

Theoretical: Enhances understanding of public MCS using a hybrid approach integrating public management frameworks and configurational analysis; supports context-specific rather than universal MCS prescriptions.

Methodological: Illustrates fsQCA's relevance for multidimensional and contextual organizational phenomena, identifying empirically valid non-linear causal paths.

Practical: Provides Moroccan public decision-makers with actionable guidance: invest in organizational culture, foster cooperation, stabilize teams, and ensure coherent alignment of responsibilities, resources, and evaluation systems.

Limitations and Perspectives

As with any study, this research has limitations. It is based on a configurational analysis at a single point in time, focusing on a sample of commercial public enterprises. Extending the research to other types of public enterprises (socially or territorially oriented, etc.) could broaden the generalizability of the findings. Additionally, the study relies on calibrated qualitative data, which, although rigorous, involves threshold choices and interpretation that may influence the results.

Future longitudinal studies could complement this research by assessing the sustainability of high-performing configurations and their ability to evolve over time. Integrating citizen-oriented indicators (user satisfaction, transparency, trust) would also help capture the intangible dimensions of public performance more effectively.

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Credit Scoring Through Mathematical Modeling: Applying the Sherrod Approach to Solaria Tech (Morocco, 2023)

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I. INTRODUCTION

Credit risk constitutes a major challenge that financial institutions face in the contemporary period, owing to its direct effects on financial stability and the profitability of banks. Credit is key to driving economic growth, but granting it carries the risk of non-payment, which can lead to significant losses that threaten the efficiency and sustainability of financial institutions. The significant expansion and complexity of financial operations has led to an urgent need for accurate tools that help measure and manage these risks effectively, away from relying exclusively on expertise. Subjectivity or personal judgment.

In this context, quantitative models, especially mathematical programming, have emerged as an effective way to formulate lending decisions within a mathematical framework that helps to reach optimal solutions, considering profit goals, regulatory constraints, and acceptable risk levels. The field of credit risk modeling has witnessed a significant development, from traditional statistical models to the use of artificial intelligence and machine learning techniques, which has contributed to improving the prediction of risk levels, although these models often lack clear mechanisms for making the optimal decision given limited resources and multiple limitations.

This study aims to develop a mathematical programming model capable of supporting credit decisions within Moroccan financial institutions, incorporating local specificities and realistic constraints. The main question of research is: How can mathematical programming be used to build an effective framework for making optimal credit decisions that minimize losses and maximize returns? To answer this question, the study seeks to achieve four main objectives: Analyze mathematical models used in credit risk assessment, build a mathematical model appropriate to the Moroccan context, integrate the results of statistical models or machine learning techniques into the mathematical model, and evaluate the performance of the proposed model in practice.

The study is based on a combination of quantitative and qualitative approaches. It begins by reviewing the literature on credit risk and the models used to evaluate it, as well as reviewing the applications of mathematical programming in the financial field. A mathematical model is then designed that considers the characteristics of the Moroccan financial system, and includes the definition of the objective function, variables, and associated constraints. Data is collected from local sources or via real-world simulations, and the model is applied using mathematical optimization tools. The results are analyzed to measure the effectiveness of the model in achieving its objectives, and discussed in the context of previous literature, with recommendations addressed to Moroccan financial institutions.

II. Literature Reviews

The study was also based on a review of a previous research by a doctoral student at the The study was also bas University of M'sila in Algeria, in which she addressed the same topic, and highlighted the importance of mathematical models in supporting banking decision making by predicting osses and reducing the likelihood of default. This underscores the growing potential importance of this area, and the need to invest in the development of accurate and effective quantitative tools in risk management.

Challenges facing commercial banks due to its direct Credit risk is one of the most prominent impact on their financial soundness and profitability. An academic study from the University of M'sila addressed this topic from a quantitative angle, focusing on the role of ting the probability of borrowers defaulting and predicting mathematical models in estimating potential loss. (Hull, 2018). 3

The study defined credit risk comprehensively, reviewing its basic types such as risk of ntration risk, and counterparty. Traditional default deterioration of creditworthiness, conceptual assessment tools such as baseline models, classification tables and early warning indicators were also presented (Bessis, 2015) (Anderson, The Credit Scoring Toolkit: Theory and Practice for Retail Credit Risk Management and Decision Automation, 2007).

The study then moved to advanced models, such as statistical models and logistic artificial intelligence regression, structural and reductionist models, to modern models based on and machine learning, because of their high accuracy in predicting risks . . (Anderson, The Credit Scoring Toolkit: Theory and Practice for Retail Credit Risk Management and Decision Automation, 2007)

study also highlighted the importance of mathematical programming in building financial The models, with an explanation of their different forms, from linear and nonlinear modeling to ing of financial dynamic and random. It showed its applications in portfolio management, pricinstruments, asset and liability analysis, and risk management of various kinds (M. Mokhtar, 2014).

However, the study did not address some recent models such as informal forests and augmented ed practical applications using real data. It also ignored the impact of global systems, and omittcrises and economic fluctuations, and did not differentiate between Islamic and conventional banks or between emerging and developed markets. (Dridi, 2011)

It also missed the organizational dimension such as the Basel standards, and the chronological evolution of the models adopted. Therefore, subsequent research should integrate quantitative models with practical applications, accounting for diverse economic and institutional contexts, to develop more precise and comprehensive credit assessment tools in the era of digital transformations. (Lieberman).

This research was based on a previous study by Riyad Mezher Abdullah and Atheer Abbas Abbadi (2022), which aimed to predict financial failure in the Bank of Baghdad using the Sherrod and Spring models. The study found that the bank suffers from clear indicators of financial weakness, and that the results fall into the category through which it is difficult to accurately predict the risk of bankruptcy, which indicates the importance of strengthening financial analysis tools and taking corrective action.

Financial failure is one of the most prominent challenges facing financial and banking institutions and is defined as the inability of an institution to meet its financial obligations when they are due. This failure does not occur suddenly but is the result of the accumulation of a set of factors and causes over a long period of time that ultimately leads to a loss of financial and operational balance (wajihad, 2022) .In the same context, financial failure is seen as the stage before the declaration of bankruptcy, as the institution has lost its ability to pay its obligations to third parties (sharife, 2022).

Researchers classified financial failure into multiple types according to its nature and dimensions. Among these types, economic failure, which occurs when a company is unable to cover its costs from the revenue generated, is an indicator of weak economic efficiency. As for financial failure, it means the inability to pay outstanding debts and obligations. There is also financial hardship, which has two types: 'apparent', when liabilities exceed the size of the assets, and 'real', when the enterprise is unable to pay even if the liabilities are equal to the assets (sharife, 2022).

The causes of financial failure are numerous and can be divided into internal and external. Internal causes include poor management, ineffective operational decision-making, accumulation of losses, and poor inventory and collection management. On the other hand, external causes are represented by volatile economic conditions, intense competition, rising exchange rates, and increasing production costs (Yahya, 2006) . (hassan, 2022) The importance 4

of this classification lies in enabling institutions to analyze the roots of the crisis and take appropriate preventive measures.

The importance of predicting financial failure lies in its ability to provide early signals to organizations to avoid collapse. It helps senior management in making corrective decisions in a timely manner, and also provides investors and creditors with the necessary information to evaluate investment and financing risks (Al-Safwani, 2020). On the other hand, regulatory and government agencies rely on such predictions to maintain financial system stability and ensure the integrity of institutional performance (Amer, 2023).

Quantitative models are among the most prominent tools used in predicting financial failure, and several models have been developed for this purpose. Most notably, the Sherrod (1981) model, which is based on six financial indicators that cover the basic aspects of financial performance, namely liquidity, profitability, and solvency, and is calculated through a Z-Score equation used to classify institutions into five categories according to the degree of risk. It is considered one of the adopted models in assessing credit risk (Lutfi, 2005). The Springate 1978 model is also one of the relatively simplified models, as it relies on four main financial ratios used to distinguish between healthy companies and those prone to failure. A company is classified as vulnerable to bankruptcy if the Z value is less than 0.96. As for the (Altman E. I., 1968) model, it is one of the leading models in this field, as it was the first to use the multiple discriminant analysis (MDA) method to predict financial failure. It incorporated five basic financial indicators into its equation, making it a widely used model in different industrial environments (Al-Kanani, 2022).

Financial indicators are vital tools that support forecasting models. Among the most prominent of these are return on assets (ROA) and return on equity (ROE), which are key indicators for measuring resource utilization efficiency. Liquidity and solvency indicators also play a pivotal role in revealing the organization's ability to meet its short and long-term obligations, contributing to objective judgments about its financial stability (Amer, 2023).

III. Methodology

Evaluating financial performance and analyzing the risk of financial failure is one of the core topics in accounting science and financial management. Among the quantitative models developed for this purpose, the Sherrod Model stands out as a diagnostic tool aimed at early prediction of companies' vulnerability to financial failure by analyzing financial ratios extracted from published financial statements (AL-Hmadane, 2023).

The Sherrod model is based on logic similar to predictive models such as Altman's Z-Score model; however, it is distinguished by its mathematical structure and the number of indicators used. The model is based on the idea that financial performance can be summarized through a set of vital ratios, which are integrated into a unified equation to produce a composite value (Z-index) used to assess the financial stability of the enterprise (Altman E. I., 1968) .

The model is based on a number of pivotal financial indicators, which often include and their formulation may vary depending on the applied study the following:

Table 1 : Causal Conditions: Synthetic Definitions

Category	Risk Classification	Z value
Grade 1	Exceptional financial health	$Z \geq 25$
Grade 2	Low risk	$25 > Z \geq 20$
Grade 3	Moderate risk	$20 > Z \geq 5$
Grade 4	High risk	$5 > Z \geq -5$
Grade 5	Critical risk	$-5 > Z$

Source: Using Altman and Sherrod Z-Score Models to Diagnose Financial Failure of Banks Listed on the Iraq Stock Exchange

Based on the results extracted from the Sherrod model shown in the table above, companies can be classified according to their level of bankruptcy risk by comparing the calculated values with predetermined criteria. This classification depends on the Z-Score calculated for each company, which reflects its financial position and degree of exposure to financial failure risk. To facilitate the interpretation and analysis process, the following table shows the classification criteria adopted in the Sherrod model and the interpretation of each category:

Table 2 :Degree of risk according to the Sherrod model

Variables	Ratio	Type	Relative Weight
X1	Net Working Capital / Total Assets	Liquidity Indicator	17
X2	Liquid Assets / Total Assets	iquidity Indicator	9
X3	Shareholders' Equity / Total Assets	Solvency or Leverage Indicator	3.5
X4	Net Profit Before Tax / Total Assets	Profitability Indicator	20
X5	Total Assets / Total Liabilities	Solvency or Leverage Indicator	1.2
X6	Shareholders' Equity / Fixed Assets	Solvency or Leverage Indicator	0.1

Source: Researcher's analysis, adapted from Fahmi Sheikh Mustafa's *Advanced Financial Analysis* (2008), p. 102

These financial ratios are integrated into a standardized linear scoring equation:

$$Z = X_1 a_1 + X_2 a_2 + X_3 a_3 + X_4 a_4 + X_5 a_5 + X_6 a_6$$

Where coefficients a_1, a_2, …, a_6 represent empirically derived weights, calibrated through statistical analysis or prior research, and are adjustable for sector-specific or regional economic conditions (Smith, 1993).

The resulting value of the equation is interpreted as follows: According to the Z-index, the financial position of the company and the probability of its failure can be determined based on its value. If the Z value is greater than 25, this indicates that the company is in a very good financial position and that the probability of failure is very low. If the Z value is between 20 and 25, this reflects the presence of medium risk signals that require careful financial monitoring. On the other hand, if the Z value falls below 20, the organization is considered highly exposed to financial distress risks, and financial restructuring may be necessary to overcome these difficulties (Altman, 1968; Altman, 1977).

Sherrod's model has been applied in a number of empirical studies, including a recent study conducted on the National Tourism Investment Company in Iraq during the period 2016–2021. The study found that the average annual Z-index was above 25, which indicates good financial stability throughout the study period, despite the economic challenges faced by the country. The model has demonstrated a high diagnostic capacity, particularly in environments where accounting data is readily available. (Yassin, 2024).

The financial analysis model is an effective tool due to its ease of application, as it relies on financial data available in traditional accounting statements such as the balance sheet and the income statement, allowing researchers and analysts to use it without the need for complex external data (Higgins, 2012) . Additionally, the model enables comparative analysis across various economic sectors, especially when financial weights are adjusted or when standardized models are used to assess performance. (Krishna G. Palepu, 2008) Moreover, financial analysis is an early warning tool that helps decision-makers identify potential problems and make strategic decisions related to financing or investment before risks escalate. (Gerald I. White, 2003).

Despite its effectiveness as a tool for evaluation and decision-making, financial analysis has several limitations that must be considered. Among them is the stability of the weights used in some models, which may not reflect the specific characteristics of all companies or sectors, leading to inaccurate results when applied generally (Krishna G. Palepu, 2008). The dependence of financial analysis on accounting statements makes them vulnerable to recognition or even manipulation, particularly in the absence of transparency or in cases of creative accounting. (Gerald I. White, 2003). Furthermore, financial analyses does not account for qualitative factors that play a crucial role in organizational performance, such as management efficiency, political stability, or organizational culture, which are difficult to quantify despite their significant impact on financial outcomes (Higgins 2012 ،) .

To enable this model, the Python programming language will be used in credit risk analysis. This analysis includes a number of algorithms, starting first with a custom model definition algorithm, which determines the relative weights of each financial indicator, along with loan classification criteria, and suggests appropriate interest rates for each rating (excellent, low-risk, medium risk, high risk, or rejected). Underlying financial indicators are then calculated, such as the ratio of working capital to total assets, the ratio of liquid assets to total assets, and other vital ratios. Each ratio is multiplied by its specific weight to get the corresponding points, and then these points are combined to derive a final sum that is used to classify the loan application and determine the proposed interest rate, according to the ranges predefined in the form. At the final stage, the algorithm produces a detailed report that includes the studied financial statements, calculated indicators, partial points, and the grand total, as well as the loan classification and the proposed interest rate.

IV. Applied Study:

Solaria Tech is a Moroccan company specialized in developing smart and sustainable solar energy solutions, founded with the aim of making clean energy accessible to everyone in North Africa. The company offers a full range of products, including smart solar panels, grid-independent home solutions, high-quality installation, and maintenance services, as well as a digital application to monitor and analyze energy consumption. The company is currently seeking funding to expand its energy storage systems business. Using highly efficient lithium batteries, as part of a strategy aimed at enhancing energy independence and keeping pace with the transition towards renewable energies.

Figure 1: Balance sheet (assest) extracted from the annual report of [Solaria Tech ,2023]

Figure2: Balance sheet (Liabilities) extracted from the annual report of [Solaria Tech ,2023]

In order to model the credit risk of Solaria Tech, we used the mathematical model "Sherrod", where the company's financial statements for the year 2023, represented in budgets and tables of results accounts :

were collected in order to calculate the indicators that make up the Sherrod model, which we have already discussed. After extracting and calculating the financial values and ratios from the budget and the table of calculating the results, we reached to:

Table 3 : Values extracted from the balance sheet and the income statement (Unit: AED)

Values	2023
Working Capital	1479926.23
Total Assets	7663646.62
Liquid assets	395107.28
Total Liabilities	7663646.62
Net Shareholders' Equity	2590436.74
Net profit before tax	727833.36
Total fixed assets	1686732.21

Source: Prepared by us with the approval of the financial statements extracted from Solaria Tech.

We observe from the table that, after extracting the financial values from the balance sheet and the income statement for the year 2023, the purpose is to calculate the indicators used in the Sherrod model, in order to assess the company's position in relation to managing the associated financial risks.

To determine the independent variables of the Sherrod model, we calculated the relevant financial ratios as indicated in Table (1) above. This was done by utilizing the data extracted from the financial statements for the years under study. The following table presents these calculations.

Table 4 :Independent Variables Resulting from the Application of the Sherrod Model

Variable	Ratio	Ratio Value	Relative Weight	Score (Points)
X1	Working Capital / Total Assets	0.19	17	3.28
X2	Cash Assets/ Total Assets	0.05	9	0.46
X3	Net Shareholders' Equity / Total Assets	0.33	3.5	1.18
X4	Net Profit Before Tax / Total Assets	0.09	20	1.89
X5	Total Assets / Total Liabilities	1	1.2	1.2
X6	Net Shareholders' Equity / Total Fixed Assets	1.53	0.1	0.15

Source: Prepared by us based on the financial statements extracted from the company's balance sheets and income statements.

V. Results and Discussion

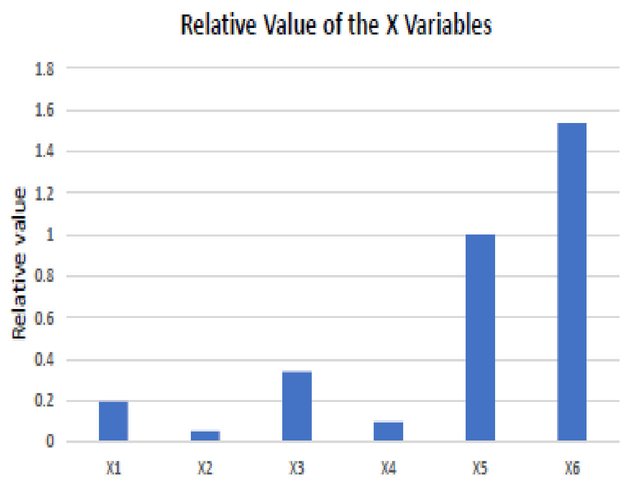

Fig1.Graph of the relative value of variables X1 to X6

Indicator 1 (X1): Working Capital to Total Assets Ratio

This indicator shows a ratio of 19.3%, meaning that approximately one-fifth of the company's assets are funded by working capital. This ratio is reasonable and indicates the company's ability to adequately finance its short-term operations. The presence of positive working capital at this level indicates that the company has sufficient liquidity to cover its current obligations without facing financial difficulties. However, the company scored only 3.28 points out of a possible 17, suggesting there is room for improvement in this indicator, possibly through better inventory management or speeding up the collection of receivables.

Indicator 2 (X2): Ratio of Cash Assets to Total Assets

The results show that liquid assets represent only 5.2% of the company's total assets, a relatively low ratio that may raise concerns about the company's immediate cash liquidity. This low ratio may reflect either weak liquidity management or a deliberate strategy to invest most funds in more productive assets. The company's score of only 0.46 points out of 9 confirms poor performance in this area. This situation may expose the company to risks related to inability to meet urgent liabilities or to take advantage of investment opportunities requiring immediate cash.

Indicator 3 (X3): Net Equity to Total Assets Ratio

The ratio of 33.8% indicates that about one-third of the company's assets are financed by equity, which is a positive indicator of the company's strong financial position and independence. This ratio reflects the company's ability to rely on its own resources to finance operations, reducing financial risks linked to excessive borrowing. This level of self-financing also provides greater flexibility in strategic decision-making without heavy reliance on external lenders. The score achieved, 1.18 points out of 3.5, reflects relatively good performance in this indicator.

Indicator 4 (X4): Ratio of Net Profit Before Tax to Total Assets

This indicator is one of the most important profitability measures as it assesses the efficiency of the company in using its assets to generate profits. A ratio of 9.5% is reasonable and indicates the company's ability to achieve an appropriate return on its investments. However, due to the high relative weight of this indicator (20 points), reflecting its importance in overall financial performance evaluation, the company's score of only 1.90 points out of 20 reveals a significant performance gap. This suggests the company needs to make additional efforts to improve asset utilization efficiency and maximize profits from these assets

Indicator 5 (X5): Ratio of Total Assets to Total Liabilities

The results show this ratio equals exactly 1.00, reflecting the company's basic accounting balance where assets equal liabilities. This balance aligns with the fundamental accounting equation stating that assets equal the sum of liabilities and equity. The company's full score for this indicator (1.20 points) reflects full compliance with accounting standards and balance in the financial structure. This indicator does not necessarily reflect strength or weakness in performance but confirms the accuracy of the presented financial statements.

Indicator 6 (X6): Ratio of Net Equity to Total Fixed Assets

A ratio of 1.536 means that equity exceeds the value of fixed assets by about 53.6%, which is a very positive sign indicating the strength of the company's financial structure. This means the company does not rely solely on external financing to fund fixed assets but has surplus equity that can be used for future growth or to withstand difficult economic conditions. This level of financial stability provides great flexibility in long-term planning and reduces financial risks associated with excessive long-term debt. The low weight of this indicator (0.1 points) means its impact on the final score is limited.

Overall Rating & Conclusion

Based on the comprehensive analysis of all financial indicators, the company attained a score of 8.28 points out of a possible 51.8, representing approximately 16%. This outcome underscores significant challenges across multiple facets of the company's financial performance, necessitating decisive and comprehensive corrective measures. The company's key strengths are evident in its solid financial independence and sufficient working capital. Conversely, the primary areas of concern include inadequate cash liquidity and a critical need to enhance profitability. Management should prioritize improving the return on assets, given its substantial weighting in the evaluation model, alongside strengthening cash liquidity to ensure the company's capacity to meet short-term obligations and leverage investment opportunities promptly.

Table 5: Loan Classification by Risk Grade and Interest Rate Determination

Year	2023
Calculated Z-Score	8.182970822
Quality Indicator	Medium Risk Loans (Category III)

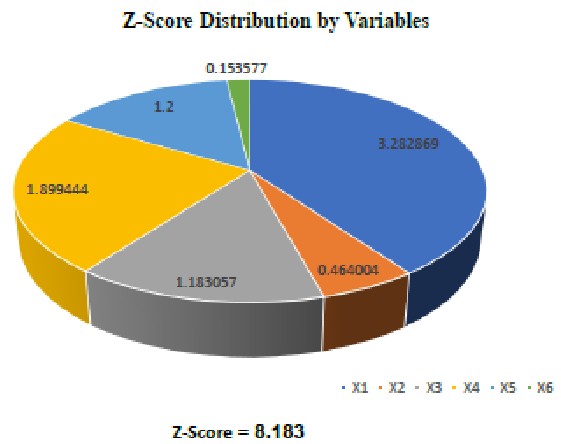

Fig2. Z-Score Distribution by Variables

When applying the Sherrod model to Solaria Tech, an active company in solar energy solutions, the 2024 financial statements were used to derive the baseline ratios for assessing credit risk. The outcome was a Z-Score of **8.28 out of a possible 51.8**, which corresponds to only **16%** of the maximum score.

By analyzing the model's indicators, it is clear that the ratio of working capital to total assets (**X1 = 19.3%)** shows that there is reasonable liquidity to enable the company to finance its short-term operations, although not at an optimal level. The ratio of cash assets (**X2 = 5.2%)** is worrying and reflects a severe weakness in spot liquidity, which may expose the company to difficulties in meeting urgent liabilities.

On the other hand, the ratio of equity to total assets (**X3 = 33.8%)** is a positive indicator that reflects good financial independence and the ability to self-finance a significant part of assets, which alleviates external financing pressure. The profit-to-asset ratio (**X4 = 9.5%)** reflects acceptable profitability performance but does not meet the high standards required by the model, making improving profitability a future priority.

As for the asset-to-liability ratio (**X5 = 1.00**), it indicates a basic accounting balance without any excess financial buffer. The fixed assets equity financing ratio (**X6 = 1.536**) is a clear strength indicating a solid capital structure and long-term independence.

Based on these indicators, **Solaria Tech's performance is mixed**; While it exhibits a sound capital foundation and self-financing ability, it suffers from liquidity constraints and moderate profitability. Despite being in the "risk zone", the company holds potential for restructuring and recovery, especially if a comprehensive reform plan is implemented. The model was executed using Python, enabling precise and swift risk simulations and quantitative assessment.

Comparing results with a previous study

When comparing the results of applying the Sherrod model to Solaria Tech with the findings of the previous study by Abdulrahman and Abadi (2022) on the Bank of Baghdad, a clear variation in the indicators achieved can be observed despite the use of the same mathematical model. While the Iraqi study indicated that there are severe difficulties in predicting bankruptcy risks due to the weakness of the banks' financial indicators, Solaria Tech shows mixed indicators combining some strengths, such as equity strength and a reasonable working capital ratio, with a clear weakness in liquidity and profitability.

This positions the company in the third risk category, where it is difficult to accurately predict the risk of financial failure.

This discrepancy highlights the ability of the Sherrod model to adapt to different economic environments. However, its effectiveness remains dependent on the quality of the input data and the nature of the sector under study. In the Iraqi case, the results emphasized solvency weaknesses and low profitability in the banking sector, whereas the Moroccan case reflects the specific characteristics of the renewable energy sector, particularly its high dependence on fixed assets, liquidity constraints, and the impact of long-term investment structures on short-term financial flexibility.

Discuss practical implications and limitations in the model:

In practice, the Sherrod model is an effective tool for supporting credit decisions, especially when used as an early warning indicator that highlights potential risk areas within an organization, without relying on it as a critical tool alone. The realistic application of this model to a company active in the renewable energy sector, such as Solaria Tech, has shown its ability to provide an accurate initial diagnosis of financial condition, allowing financial institutions to adopt financing decisions conditional on realistic reform plans. However, the effectiveness of the model remains limited with a set of constraints, most notably the stability of the weights used, which may not reflect local or sectoral specificities, which necessitates the need to adapt them to the Moroccan context. The model also relies on accounting data, which makes it vulnerable to distortions resulting from self-assessment or poor transparency, in addition to ignoring fundamental qualitative factors such as management efficiency and political stability. Among other limitations, the model's ability to monitor temporal and dynamic changes is weak, providing only a real-time picture without tracking future developments. From this standpoint, the study recommends the need to recalibrate the weights to suit the Moroccan economic environment, or to work on integrating the Sherrod model with modern technologies such as artificial intelligence or neural networks, in order to enhance its predictive accuracy and reduce the exclusive reliance on accounting data.

VI. Conclusion

The results of applying the Sherrod model to the company's financial statements for the year 2024 indicate a deep structural imbalance in financial performance that requires radical and urgent intervention from senior management. The overall score of 8.28 points out of a possible 51.8, which is equivalent to only 16% of the grand total, puts the company in the area of acute financial risk and calls for a radical reconsideration of all current financial, operational and management strategies. This low level of performance indicates that the company faces multiple challenges Dimensions require comprehensive and integrated solutions. The company's relative financial independence, reflected in the equity ratio of 33.8% of total assets, is the lifeblood and the only strategic strength on which to build in the process of comprehensive financial reform. This level of self-financing provides the company with vital flexibility in making radical reform decisions without being subjected to excessive external pressure from lenders or financial institutions. This independence also gives management greater freedom to implement long-term strategies that may require short-term sacrifices. For sustainable gains. While the acute and serious weakness in cash liquidity is manifested in the fact that liquid assets represent only 5.2% of total assets, this is an explicit red alert for imminent and potential problems in the company's ability to meet its short-term obligations. This low level of liquidity affects not only day-to-day operations, but also the company's ability to exploit urgent investment opportunities and deal with sudden financial crises. Getting only 0.46 points out of 9 points in this indicator Confirms the magnitude of the risk faced by the company in this vital aspect. 14 The discrepancy between the theoretically reasonable 9.5% return on assets ratio and the low score achieved in this index (1.90 out of 20) reveals a huge gap between current performance and the required standards of financial excellence. This discrepancy clearly indicates that the criteria of the Sherrod model set a high bar for outstanding performance, and that the company needs to make exceptional efforts to achieve the required levels of profitability. The high relative weight of this indicator (20 points) makes improving it a top priority that cannot be postponed. Other indicators reveal a mixed picture that combines relative strengths with areas in need of substantial improvement. Positive working capital of 19.3% provides a reasonable safety cushion for operations, but the lack of full scores in this indicator indicates that operational cycle management can be improved. The sixth indicator related to the ratio of equity to fixed assets shows a very positive position of 1.536, reflecting the strength of the long-term financial structure and the stability of the funding base. This comprehensive analysis reveals that the company stands at a critical crossroads, having the solid financial foundations for reform and growth, but suffering from severe operational and management problems that hinder the optimal exploitation of this potential. Success in dealing with these challenges will determine the company's future path and its ability to achieve sustainable growth and high profitability.

Considering the shocking and disturbing results revealed by the implementation of the Sherrod model, the company urgently needs to develop and implement a comprehensive, multi-phased reform plan over a period of at least five years to achieve a radical and tangible improvement in all financial and operational performance indicators. This plan must be bold, comprehensive, and realistic at the same time, considering the available possibilities and limitations, and most importantly, making the most of existing strengths to reinforce weaknesses. Where in the first stage Most urgently, it should focus intensively and focused on addressing the acute liquidity crisis through a comprehensive strategy to restructure the asset portfolio and improve the efficiency of working capital management. The primary and vital goal is to raise the ratio of cash assets from the current dangerous level of 5.2% to a safe level of 12-15% of total assets over the next twelve months. This drastic improvement in liquidity will provide the company with the necessary financial security and vital flexibility to face urgent obligations and exploit profitable investment opportunities that may suddenly appear in the market. To achieve this ambitious goal, it requires the implementation of an integrated set of practical and immediate measures, including primarily accelerating the cycle of collection of receivables through the application of stricter credit policies and improving monitoring and collection systems. Inventory management needs to be reassessed and improved to reduce frozen capital in slow-moving goods, with modern inventory management systems in place that ensure the availability of essential commodities without overstocking. In addition, the sale of some non-core assets could be considered or unproductive to free up cash without compromising the company's core operations. After the implementation of the phase, we move on to the second and longer-term phase that requires an intensive strategic focus on radically improving profitability through a comprehensive and in-depth review of all aspects of the cost structure and improving the efficiency of operations at all levels. The strategic objective is to raise the return on assets from the current level of 9.5% to a distinct level of 15-18% within the next two years, which will lead to a tripling of the points achieved in this vital indicator at least. This significant improvement will have a tremendous positive impact on the overall score of the company due to the high relative weight of this indicator in the Sherrod model. In order to achieve this fundamental improvement in profitability, it requires the implementation of a multidimensional strategy that includes smart and thoughtful investment in modern technology and advanced systems that increase production efficiency and reduce 15 operational costs. Supply chains must be redesigned and optimized to reduce costs and improve product quality, as well as develop new and innovative products and services with higher profit margins that meet evolving market needs. Investing in training and developing human resources will be a crucial factor in the success of this phase, as improving the skills of employees will reflect positively on productivity and quality. In the third and final phase, a company should focus on making the best use of the improvements achieved in the previous two phases to achieve sustainable growth and thoughtful strategic expansion. The core strength of strong financial independence and high equity ratio requires smart exploitation through the application of a balanced investment strategy that achieves ambitious growth without sacrificing carefully built financial stability. The company at this stage would be ideally positioned to invest in long-term and profitable projects without the need for excessive borrowing or exposure to unforeseen financial risks. Developing a comprehensive strategy to diversify the business and expand the production and service base will significantly help in sustainably improving the return on assets and reducing operational risk by not relying on a single sector or product. This diversification should be thoughtful and based on comprehensive feasibility studies and in-depth analysis of target markets. Investing in R&D will be a critical factor in ensuring long-term competitiveness and developing innovative products and services that generate high profit margins. Success in achieving these ambitious goals requires the application of a set of supportive and enhanced strategies that ensure continuous improvement and avoid setbacks, through continuous and accurate monitoring of all financial performance indicators and the application of the Sherrod model on a quarterly basis that will ensure regular and objective evaluation of the effectiveness of the actions taken and the possibility of rapid and flexible adjustment of plans and strategies as needed and changing circumstances. In addition to developing a management information system that will provide the department with accurate data and deep analysis necessary to take Decisions based on scientific and objective foundations. This system should include key performance indicators, advanced financial analytics, and future forecasts based on sophisticated statistical and mathematical models. An organizational culture that focuses on continuous improvement and innovation must be developed, while motivating employees to actively participate in the development and reform process. The ultimate and ambitious strategic goal remains to reach 50-60% of the total points in the Sherrod model over the next five years, which will move the company from an acute financial risk zone to an outstanding and globally accepted level of performance and establish sustainable and profitable growth in the long term. This radical improvement will position the company in a strong competitive position in the market and open new opportunities for local, regional, and international growth and expansion. The implementation of this ambitious vision will require the full commitment of all levels of management and staff, with the need to provide Financial and human resources necessary for the successful implementation of the plan. Success in this major challenge will transform the company from an organization with serious financial problems to a role model in financial and operational excellence, which will benefit all stakeholders from shareholders, employees, customers, and business partners

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Optimality Conditions for Interval-Valued Optimization Problems: A Real-Valued Optimization Approach

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***Abstract*—This paper explores a class of interval-valued optimization problems (IVOPs). An associated scalar optimization problem (ASOP) and a constrained optimization problem (ACOP) are formulated associated with IVOP. It is demonstrated that any optimal solution of either ASOP or ACOP qualifies as an LU-solution of IVOP. Additionally, sufficient optimality conditions for IVOP are derived under strong convexity assumptions on the objective function. To illustrate the practical significance of these conditions, they are applied to solving complex IVOPs using MATLAB R2024a.**

***Index Terms*—Interval-valued optimization, Optimality conditions, Generalized Hukuhara derivative, Constraint optimization problems**

I. Introduction

It has been observed that many optimization methods existing in the literature assume precise values for objective functions and constraints (see, [1], [9]). However, real-world situations often involve inherent uncertainties in data or model parameters, rendering classical optimization techniques inadequate for such models (see, [4], [18]). Consequently, various probabilistic and non-probabilistic approaches, such as stochastic programming [3] and robust optimization [2], have been developed to address these uncertainties. However, these methods have their drawbacks. In stochastic programming, uncertain coefficients are treated as random variables with known distribution functions, which can be challenging to determine in critical scenarios. On the other hand, robust optimization always considers the worst-case scenario, potentially leading to overly conservative and costly solutions when favorable conditions arise.

Recently, interval analysis has emerged as a non-probabilistic approach to address imprecise information in models, where the lower and upper bounds of uncertain parameters are estimated from historical data. Building on this foundation, interval-valued optimization problems (IVOPs) provide a robust framework for managing such uncertainties by representing objectives and constraints with intervals rather than fixed values. This approach significantly enhances the applicability of IVOPs in practical optimization scenarios such as engineering and management sciences (see, [14], [16]). The initial development and formulation of IVOPs are credited to Ishibuchi and Tanaka [6], where the authors studied an IVOP by formulating a corresponding multiobjective optimization problem. Wu [15] presented sufficient optimality conditions for IVOP under Hukuhara differentiability and convexity assumptions. Further, Zhang et al. [17] employed the concept of invexity to establish sufficient optimality conditions for IVOP involving differentiable functions. Rahman et al. [11] explored necessary and sufficient optimality conditions for IVOPs, involving differentiable lower-bound and upper-bound functions of the objective function of IVOP. On the other hand, Osuna-Gómez et al. [10] established necessary optimality conditions for IVOPs involving generalized Hukuhara differentiable objective function, which is defined on a subset of $\mathbb{R}$.

Recently, interval analysis has emerged as a non-probabilistic approach for addressing imprecise information in models, in which the lower and upper bounds of uncertain parameters are estimated from historical data. Building on this foundation, interval-valued optimization problems (IVOPs) provide a robust framework for managing such uncertainties by representing objectives and constraints with intervals rather than fixed values. This approach significantly enhances the applicability of IVOPs in practical optimization scenarios such as engineering and management sciences (see [14], [16]).

It is worthwhile to note that optimality conditions for IVOPs have been investigated by many researchers by employing the Hukuhara derivatives (see, Wu [15]) and differentiability of upper-bound and lower-bound functions of interval-valued function (see, Rahman et al. [11]). However, optimality conditions for IVOP using the notions of gH-differentiability have only been investigated when the domain of the objective function is a subset of real numbers (see, Osuna-Gómez et al. [10]). In this article, the primary objective is to introduce sufficient optimality conditions for IVOP in terms of ACOP and ASOP, enabling the use of well-developed constrained and unconstrained optimization methods for real-valued functions to solve IVOP. Necessary optimality conditions for IVOP are established by leveraging gH-differentiability. Additionally, sufficient optimality conditions are derived by imposing an additional assumption of strong convexity on the objective function of IVOP.

Motivated by the works of [10], [11], [15], this article studied a class of IVOP in the context of Euclidean spaces. For the proposed IVOP, associated scalar-valued optimization problems (ASOP) and constrained optimization problems (ACOP) are illustrated. It is shown that the optimal solutions of either ASOP or ACOP correspond to the LU-solution of IVOP. Moreover, necessary optimality conditions for the existence of the LU-solution of IVOP are established under generalized partial Hukuhara differentiability assumptions on the objective function. Sufficient optimality conditions for IVOP are also derived under strong convexity hypotheses. For illustrative purposes, the results are complemented with numerical examples.

The novelty and contributions of this article are discussed as follows. The approach of converting IVOP to ASOP and ACOP enables us to solve IVOP by utilizing well-established numerical techniques designed for addressing both constrained and unconstrained optimization problems of real-valued functions. Furthermore, necessary and sufficient optimality conditions for IVOP are established under appropriate assumptions. The results presented in this paper extend and generalize several significant findings already existing in the literature. Specifically, the optimality conditions for IVOP are real numbers established by Osuna-Gómez et al. [10] has been extended to the optimality condition for IVOP on $\mathbb{R}^n$.

The structure of the paper is outlined as follows. Section II provides a review of fundamental definitions, notations, and terminologies that are essential for subsequent discussions. Section III focuses on establishing the optimality conditions for IVOP and is divided into two parts. In Subsection III-A, ASOP and ACOP corresponding to IVOP are introduced, and the relationship between the solutions of ACOP and the effective solutions of IVOP is explored. To illustrate the practical applicability of these results, several non-trivial numerical examples are presented. In Subsection III-B, the necessary optimality conditions for IVOP are established using gH-differentiability, and sufficient optimality conditions are derived under the additional assumption of strong convexity of the objective function. Finally, Section V concludes the paper by summarizing the main findings and suggesting potential directions for future research.

II. Notations and Mathematical Preliminaries

III. Notations and Mathematical Preliminaries

The symbol $\mathbb{R}^k$ and $\mathbb{N}$ will be used to denote the k-dimensional Euclidean space and the set of all natural numbers, respectively. For $l \in \mathbb{N}$, the set $\{1, 2, \ldots, l\}$ will be denoted by $\mathcal{E}_l$. For any set $\mathcal{G}$, the symbol $\mathcal{G}^k$ will be defined as follows:

$$\mathcal{G}^k := \underbrace{\mathcal{G} \times \cdots \times \mathcal{G}}_{k \text{ times}}.$$

Unless explicitly mentioned, the symbols $\mathcal{Z}$ and $\mathcal{S}$ will be used to denote non-empty open subsets of $\mathbb{R}^n$ and $\mathbb{R}$, respectively. The following definition is from [**?**].

Definition II.1 Let

$$\Upsilon$$

:S $\rightarrow \mathbb{R}$ be a real-valued function and $\hat{a} \in \mathcal{S}$. Then, the lateral derivatives of T at the point $\hat{a}$ are defined as follows:

$$\Upsilon'_+(\hat{a}) := \lim_{\beta \to 0^+} \frac{\Upsilon(\hat{a}+\beta) - \Upsilon(\hat{a})}{\beta},$$

and $\Upsilon'_-(\hat{a}) := \lim_{\beta \to 0^-} \frac{\Upsilon(\hat{a}+\beta)-\Upsilon(\hat{a})}{\beta}$, provided both the limits exist.

The set $\{[\underline{u}, \overline{u}] : \underline{u}, \overline{u} \in \mathbb{R} \text{ and } \underline{u} \leq \overline{u}\}$ referred to as $\mathcal{I}(\mathbb{R})$. The collection of all the elements $[\underline{u}, \overline{u}] \in \mathcal{I}(\mathbb{R})$, satisfying $\underline{u} \leq \overline{u}$ is denoted by $\mathcal{I}(\mathbb{R})$. For $\xi_1, \xi_2 \in \mathbb{R}$, the symbol $[\xi_1 \vee \xi_2]$ is employed to denote the interval $[\min\{\xi_1, \xi_2\}, \max\{\xi_1, \xi_2\}]$. For any real number u, the interval $[u, u]$ is referred to as a degenerate interval, and vice versa.

Let $\mathcal{U} = (u_1, u_2, \ldots, u_k) \in \mathbb{R}^k$, then we say that

$$\mathcal{U} \in [\underline{z}, \overline{z}] := [z_1, z_2, \ldots, z_k] \in \mathcal{I}(\mathbb{R})^k$$

if and only if $u_j \in [\underline{z}_j, \overline{z}_j]$ for every $j \in \mathcal{E}_k$.

The interval-valued function $\Upsilon : \mathcal{Z} \rightarrow \mathcal{I}(\mathbb{R})$ is defined as:

$$\Upsilon(z) := [\underline{\Upsilon}(z), \overline{\Upsilon}(z)], \qquad \text{for every } z \in \mathcal{Z},$$

where $\mathcal{Z}$ is a non-empty subset of $\mathbb{R}^n$. The functions $\underline{\Upsilon}$ and $\overline{\Upsilon}$ are real-valued functions referred to as lower-bound and upper-bound functions of Υ, respectively.

Let $\mathcal{W} := [\underline{w}, \overline{w}]$, $\mathcal{X} := [\underline{x}, \overline{x}] \in \mathcal{I}(\mathbb{R})$ be arbitrary. The following algebraic operations from [**?**] will be used in the sequel:

$$\mathcal{W} \oplus \mathcal{X} := \{w + x : w \in \mathcal{W},\, x \in \mathcal{X}\} = [\underline{w} + \underline{x},\, \overline{w} + \overline{x}],$$

$\mathcal{W} \ominus \mathcal{X} := \{w - x : w \in \mathcal{W},\, x \in \mathcal{X}\} = [\underline{w} - \overline{x},\, \overline{w} - \underline{x}].$

$$\text{k} \odot \mathcal{W} := \{kw : w \in \mathcal{W}\} = \begin{cases} [k\underline{w},\, k\overline{w}], & k \geq 0, \\ [k\overline{w},\, k\underline{w}], & k \leq 0. \end{cases}$$

We recall the following ordered relations from [?]. Let $\mathcal{W} := [\underline{w}, \overline{w}]$, $\mathcal{X} := [\underline{x}, \overline{x}] \in \mathcal{I}(\mathbb{R})$. The ordered relations between $\mathcal{W}$ and $\mathcal{X}$ are defined as follows:

$$\mathcal{W} \leq_{LU} \mathcal{X} \iff \underline{w} \leq \underline{x} \text{ and } \overline{w} \leq \overline{x},$$

$$\mathcal{W} <_{LU} \mathcal{X} \iff \mathcal{W} \leq_{LU} \mathcal{X} \text{ and } \mathcal{W} \neq \mathcal{X},$$

$$\mathcal{W} <_{LU} \mathcal{X} \iff \underline{w} < \underline{x} \text{ and } \overline{w} < \overline{x}.$$

Let $\mathcal{W}^k = (\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_k)$, $\mathcal{X}^k = (\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_k) \in (\mathcal{I}(\mathbb{R}))^k$, then the ordered relations between $\mathcal{W}$ and $\mathcal{X}$ are defined as follows:

$$\mathcal{W}^k \leq_k \mathcal{X}^k \iff \mathcal{W}_i \leq_{LU} \mathcal{X}_i, \quad \text{for every } i \in \mathcal{E}_k,$$

$$\mathcal{W}^k <_k \mathcal{X}^k \iff \mathcal{W}_i <_{LU} \mathcal{X}_i, \quad \text{for every } i \in \mathcal{E}_k.$$

The following definitions are from Wu [?].

Definition II.2 The gH-difference between $\mathcal{W} := [\underline{w}, \overline{w}]$ and $\mathcal{X} := [\underline{x}, \overline{x}] \in \mathcal{I}(\mathbb{R})$, denoted by $\mathcal{W} \ominus_{gH} \mathcal{X}$, is defined as follows:

$$\mathcal{W} \ominus_{gH} \mathcal{X} := [\min\{\underline{w} - \underline{x},\, \overline{w} - \overline{x}\},\, \max\{\underline{w} - \underline{x},\, \overline{w} - \overline{x}\}].$$

Definition II.3 Let $\mathcal{Z} \subseteq \mathbb{R}^n$ be a convex set. The function $\Upsilon : \mathcal{Z} \to \mathcal{I}(\mathbb{R})$ is termed as a convex function if, for any $z_1, z_2 \in \mathcal{Z}$ and $\beta \in [0, 1]$, the following inequality is satisfied:

$$\Upsilon((1 - \beta)z_1 + \beta z_2) \leq_{LU} (1 - \beta) \odot \Upsilon(z_1) \oplus \beta \odot \Upsilon(z_2).$$

The following definition revisits the concept of strongly convex interval-valued functions.

Definition II.4 Let $\mathcal{Z} \subseteq \mathbb{R}^n$ be convex. The function $\Upsilon : \mathcal{Z} \to \mathcal{I}(\mathbb{R})$ is said to be strongly convex on $\mathcal{Z}$, if there exists $\gamma > 0$
such that for all $z_1 \neq z_2 \in \mathcal{Z}$ and $\beta \in (0, 1)$, the following inequality is satisfied:

$$\begin{aligned} \Upsilon\big((1 - \beta)z_1 + \beta z_2\big) &\oplus \beta \|z_2 - z_1\|^2 \odot [0, \gamma] \\ &\leq_{LU} (1 - \beta) \odot \Upsilon(z_1) \oplus \beta \odot \Upsilon(z_2). \end{aligned}$$

We recall the following definition and theorem from Stefanini and Bede [?].

Definition II.5 Let $\hat{a} \in \mathcal{S}$ and let $\Upsilon : \mathcal{S} \to \mathcal{I}(\mathbb{R})$ be an interval-valued function. The gH-derivative of Υ at $\hat{a}$, denoted by $\Upsilon'_{gH}(\hat{a})$, is defined as follows:

$$\Upsilon'_{gH}(\hat{a}) := \lim_{\beta \to 0} \frac{\Upsilon(\hat{a} + \beta) \ominus_{gH} \Upsilon(\hat{a})}{\beta},$$

provided the limit exists.

Definition II.5 Let $\hat{a} \in \mathcal{S}$ and let $\Upsilon : \mathcal{S} \to \mathcal{I}(\mathbb{R})$ be an interval-valued function. The gH-derivative of Υ at $\hat{a}$, denoted by $\Upsilon'_{gH}(\hat{a})$, is defined as follows:

$$\Upsilon'_{gH}(\hat{a}) := \lim_{\beta \to 0} \frac{\Upsilon(\hat{a} + \beta) \ominus_{gH} \Upsilon(\hat{a})}{\beta},$$

provided the limit exists.

Theorem II.1 Let $\Upsilon : \mathcal{S} \to \mathcal{I}(\mathbb{R})$ be an interval-valued function defined by:

$$\Upsilon(z) := [\underline{\Upsilon}(z),\, \overline{\Upsilon}(z)], \qquad \text{for every } z \in \mathcal{S}.$$

The function Υ is gH-differentiable at $\hat{a} \in \mathcal{S}$ if and only if one of the following holds:

(a)

1) $\underline{\Upsilon}$ and $\overline{\Upsilon}$ are differentiable at $\hat{a}$ and
$$\Upsilon'_{gH}(\hat{a}) = \left[\min\{\underline{\Upsilon}'(\hat{a}),\, \overline{\Upsilon}'(\hat{a})\}, \max\{\underline{\Upsilon}'(\hat{a}),\, \overline{\Upsilon}'(\hat{a})\}\right].$$

2) The left and right hand derivatives of $\underline{\Upsilon}$ and $\overline{\Upsilon}$ at $\hat{a}$, that is, $\underline{\Upsilon}'_-(\hat{a})$, $\underline{\Upsilon}'_+(\hat{a})$, $\overline{\Upsilon}'_-(\hat{a})$, $\overline{\Upsilon}'_+(\hat{a})$, exist and satisfy
$$\underline{\Upsilon}'_-(\hat{a}) = \overline{\Upsilon}'_-(\hat{a}), \qquad \underline{\Upsilon}'_+(\hat{a}) = \overline{\Upsilon}'_+(\hat{a}).$$
Moreover,
$$\Upsilon'_{gH}(\hat{a}) = \left[\min\{\underline{\Upsilon}'_+(\hat{a}),\, \overline{\Upsilon}'_+(\hat{a})\}, \max\{\underline{\Upsilon}'_+(\hat{a}),\, \overline{\Upsilon}'_+(\hat{a})\}\right].$$

The following definitions are from Stefanini Arana-Jiménez [?]. **Definition II.6** Let $\hat{a} := (\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_n) \in \mathcal{Z}$ and let $\Upsilon : \mathcal{Z} \to \mathcal{I}(\mathbb{R})$ be an interval-valued function. For a fixed $i \in \mathcal{E}_n$, the i^{th} partial generalized Hukuhara derivative (abbreviated as partial gH-derivative), denoted by $\dfrac{\partial_{gH}\Upsilon(\hat{a})}{\partial \hat{a}_i}$, is defined as follows:

$$\frac{\partial_{gH}\Upsilon(\hat{a})}{\partial \hat{a}_i} := \lim_{\beta \to 0} \frac{\Upsilon(\hat{a}_1, \ldots, \hat{a}_i + \beta, \ldots, \hat{a}_n) \ominus_{gH} \Upsilon(\hat{a}_1, \ldots, \hat{a}_i, \ldots, \hat{a}_n)}{\beta}$$

provided the limit exists.

If $\dfrac{\partial_{gH}\Upsilon(\hat{a})}{\partial \hat{a}_i}$ exists for all $i \in \mathcal{E}_n$, then the gH-gradient of Υ at $\hat{a}$ is defined as follows:

$$\nabla_{gH}\Upsilon(\hat{a}) := \left(\frac{\partial_{gH}\Upsilon(\hat{a})}{\partial \hat{a}_1}, \frac{\partial_{gH}\Upsilon(\hat{a})}{\partial \hat{a}_2}, \ldots, \frac{\partial_{gH}\Upsilon(\hat{a})}{\partial \hat{a}_n}\right).$$

Definition II.7 Let $\Upsilon : \mathcal{Z} \to \mathcal{I}(\mathbb{R})$ be an interval-valued function and $\hat{a} \in \mathcal{Z}$. The gH-directional derivative of Υ in the direction $d \in \mathbb{R}^n$, denoted by $D_{gH}\Upsilon(\hat{a}, d)$, is defined as follows:

$$D_{gH}\Upsilon(\hat{a}, d) := \lim_{h \to 0} \frac{\Upsilon(\hat{a} + hd) \ominus_{gH} \Upsilon(\hat{a})}{h},$$

provided the limit exists.

We now consider the following scalar optimization problem:

$$\text{(SOP)} \qquad \text{Minimize} \quad \Lambda(z),$$

$$\text{subject to} \quad z \in \mathcal{Z},$$

where $\Lambda : \mathcal{Z} \to \mathbb{R}$ represents a real-valued function. In the following definition, we recall the notion of a minimizer for the problem SOP (see, Mishra and Upadhyay [**?**]).

Definition II.8 An element $\hat{z} \in \mathcal{Z}$ is termed as a global minimizer of the problem SOP, if the following inequality is satisfied:

$$\Lambda(\hat{z}) \leq \Lambda(z), \quad \text{for every } z \in \mathcal{Z}. \tag{II.1}$$

Furthermore, if for every $z \in \mathcal{Z}$ and $z \neq \hat{z}$, the following holds:

$$\Lambda(\hat{z}) < \Lambda(z), \tag{II.2}$$

then $\hat{z}$ is called strict global minimizer of SOP. If there exists a non-empty open subset $\mathcal{N}$ of $\mathbb{R}^n$, such that (II.1) and (II.2) are satisfied for all $z \in \mathcal{Z} \cap \mathcal{N}$, then the point $\hat{z}$ is referred to as a local minimizer and a strict local minimizer of SOP, respectively.

IV. Optimality Conditions for IVOP

This section delves into the optimality conditions for IVOP and is divided into two subsections. In the first subsection, sufficient optimality conditions for IVOP are developed in terms of constrained and unconstrained real-valued optimization problems. This formulation facilitates the use of well-established optimization techniques for real-valued functions, making the solution of IVOP more effective and practical. In the second subsection, necessary and sufficient optimality conditions are established by employing the gH-derivative. This approach extends the existing necessary optimality conditions found in the literature, offering a more comprehensive framework for tackling IVOP. Together, these methodologies aim to bridge the theoretical and practical gap between interval-valued and real-valued optimization techniques, enhancing both the theoretical understanding and the practical applicability of IVOP solutions.

The interval-valued optimization problem under consideration is formulated as follows:

$$\text{(IVOP)} \qquad \text{Minimize} \quad \Upsilon(z),$$

$$\text{subject to} \quad z \in \mathcal{Z},$$

where $\Upsilon : \mathcal{Z} \to \mathcal{I}(\mathbb{R})$ is defined as:

$$\Upsilon(z) := [\underline{\Upsilon}(z), \overline{\Upsilon}(z)], \qquad \text{for every } z \in \mathcal{Z},$$

where $\underline{\Upsilon}$ and $\overline{\Upsilon}$ defined on $\mathcal{Z}$ are referred to as lower-bound and upper-bound functions of Υ, respectively. The following definition of LU-solution of (IVOP) is from Ishibuchi, H, and Tanaka [**?**].

Definition III.1 An element $\hat{z} \in \mathcal{Z}$ is said to be a global LU-solution of IVOP if the following inequality is not satisfied for any $z \in \mathcal{Z}$:

$$\Upsilon(z) \leq_{LU} \Upsilon(\hat{z}). \tag{III.1}$$

A point $\hat{z}$ is called a local LU-solution of IVOP, if there exists a neighbourhood $\mathcal{N}$ of $\hat{z}$, such that (III.1) is not satisfied for any $z \in \mathcal{Z} \cap \mathcal{N}$.

A. Optimality conditions for IVOP: A real-valued optimization approach

In this subsection, two real-valued optimization problems associated with IVOP are introduced. Additionally, the relationship between the optimal solution of IVOP and the optimal solutions of these associated problems is established.

First, corresponding to the IVOP discussed earlier, an associated scalar-valued optimization problem (ASOP) is formulated as follows:

$$\text{(ASOP)} \quad \text{Minimize} \quad \Lambda(z, \mu) := \underline{\Upsilon}(z) + \mu\big(\overline{\Upsilon}(z) - \underline{\Upsilon}(z)\big),$$

$$\text{subject to} \quad (z, \mu) \in \mathcal{Z} \times [0, 1],$$

where $\Lambda : \mathcal{Z} \times [0, 1] \to \mathbb{R}$ is a real-valued function. The following theorem establishes a relationship between the minimizer of the ASOP and the LU-solution of the IVOP.

Theorem III.1 Let $(\hat{z}, \hat{\mu}) \in \mathcal{Z} \times [0, 1]$ be a minimizer of (ASOP). If $\hat{\mu} \in (0, 1)$ then $\hat{z} \in \mathcal{Z}$ is an LU-solution of the (IVOP).

Proof. From the provided hypotheses, $(\hat{z}, \hat{\mu}) \in \mathcal{Z} \times (0, 1)$ is a minimizer of (ASOP). Therefore, for any $(z, \mu) \in \mathcal{Z} \times [0, 1]$,

$$\Lambda(\hat{z}, \hat{\mu}) \leq \Lambda(z, \mu).$$

In particular, when $\mu = \hat{\mu}$, it follows that

$$\Lambda(\hat{z}, \hat{\mu}) \leq \Lambda(z, \hat{\mu}), \qquad \text{for every } z \in \mathcal{Z}. \tag{III.2}$$

On the contrary, let us assume that $\hat{z} \in \mathcal{Z}$ is not an LU-solution of the problem IVOP. Then there exists an element $z \in \mathcal{Z}$, for which the following inequality holds:

$$\Upsilon(z) \leq_{LU} \Upsilon(\hat{z}).$$

From the above inequality, it follows that:

$$\underline{\Upsilon}(z) \leq \underline{\Upsilon}(\hat{z}) \quad \text{and} \quad \overline{\Upsilon}(z) \leq \overline{\Upsilon}(\hat{z}),$$

or

$$\overline{\Upsilon}(z) \leq \overline{\Upsilon}(\hat{z}) \quad \text{and} \quad \underline{\Upsilon}(z) \leq \underline{\Upsilon}(\hat{z}).$$

Therefore, we arrive at the following:

$$\Lambda(z, \hat{\mu}) < \Lambda(\hat{z}, \hat{\mu}),$$

which contradicts (III.2). This completes the proof.

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Corollary 1 Let $(z, \mu) \in \mathcal{Z} \times [0, 1]$ is a local minimizer of the problem ASOP. If $\mu \in (0, 1)$, then $\hat{z} \in \mathcal{Z}$ is a local LU-solution of the problem IVOP. *Proof.* The proof follows along similar lines as the proof of Theorem III.1.

Remark III.1 If $\hat{z}$ is a local LU-solution of IVOP, then there is no guarantee that we can always get $\hat{\mu} \in [0, 1]$, such that $(\hat{z}, \hat{\mu})$ will be a local minimizer of (ASOP). This fact is illustrated in the following example.

Example III.1 Consider the following interval-valued optimization problem:

$$\text{(PIII.1.1)} \qquad \text{Minimize} \quad \Upsilon(z_1, z_2),$$

$$\text{subject to} \quad (z_1, z_2) \in \mathcal{Z} := [-1, 1] \times [-1, 1],$$

where the function $\Upsilon : \mathcal{Z} \to \mathcal{I}(\mathbb{R})$ is defined as follows:

$$\Upsilon(z_1, z_2) := [\cos(z_1+z_2),\ \sin(z_1+z_2)+2], \quad \text{for all } (z_1, z_2$$

The corresponding associated real-valued optimization problem is formulated in the following manner:

$$\text{(PIII.1.2)} \qquad \text{Minimize} \quad \Lambda((z_1, z_2), \mu) = (1-\mu)\cos(z_1+z_2)+\mu\big(\sin(z_1+z_2)+2\big),$$

$$\text{subject to} \quad ((z_1, z_2), \mu) \in \mathcal{Z} \times [0, 1].$$

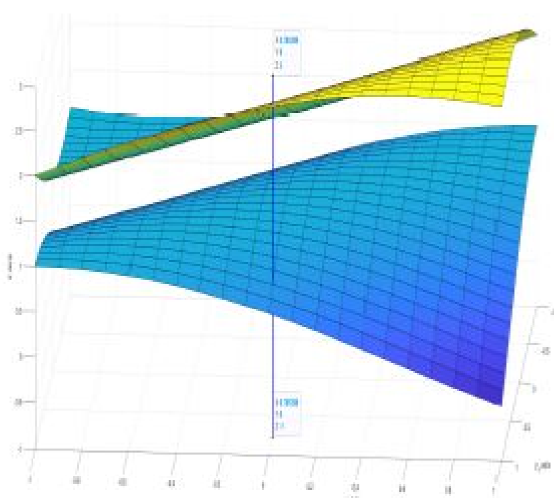

Fig. 1. $\Upsilon(x_1, x_2)$

It can be easily observed from Figure 1(a) that $\hat{z} = \left(\frac{\pi}{4}, 0\right)$ is a local LU-solution to the problem P3.1.1. However, as seen in Figure 1(b), it can be observed that $(\hat{z}, \hat{\mu})$ is not a local minimizer of P3.1.2 for any choice of $\hat{\mu} \in [0, 1]$.

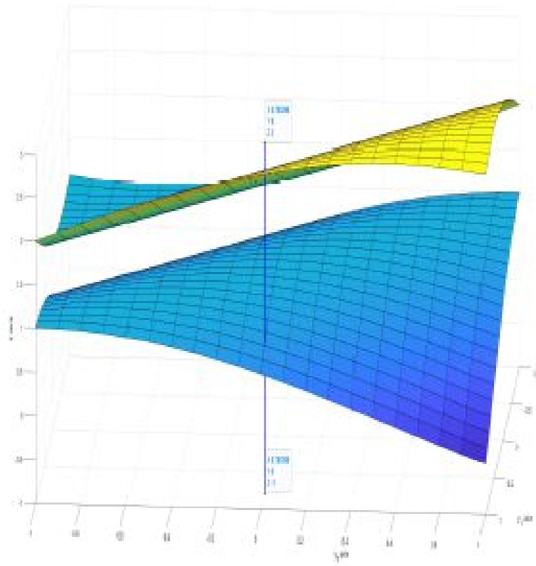

Fig. 2. $\Upsilon(x_1, x_2)$

Let $\mu \in [0, 1]$ be fixed. Now, corresponding to IVOP, we formulate another associated real-valued optimization problem ASOP1 as follows:

$$\text{(ASOP1)} \qquad \text{Minimize} \quad \Lambda(z) := \underline{\Upsilon}(z)+\mu\big(\overline{\Upsilon}(z)-\underline{\Upsilon}(z)\big),$$

$$\text{subject to} \quad z \in \mathcal{Z},$$

where $\Lambda : \mathcal{Z} \to \mathbb{R}$ is a real-valued function.

Remark III.2 It is worth noting that when $\mu = 0$ or $\mu = 1$, a local minimizer of ASOP1 may not necessarily be a local LU-solution of IVOP. To illustrate this fact, we furnish the following example.

Example III.2 Consider the following interval-valued optimization problem:

$$\text{(PIII.2.1)} \qquad \text{Minimize} \quad \Upsilon(z_1, z_2),$$

$$\text{subject to} \quad (z_1, z_2) \in \mathcal{Z},$$

where $\mathcal{Z} := (-1, 1) \times (-1, 1)$ and $\Upsilon : \mathcal{Z} \to \mathcal{I}(\mathbb{R})$ is defined as follows:

$$\Upsilon(z_1, z_2) := [\underline{\Upsilon}(z_1, z_2),\ \overline{\Upsilon}(z_1, z_2)] = \begin{cases} [0,\ z_1^2 + z_2^2], & \text{if } z_1 \leq 0, \\ [-(z_1 + z_2),\ 2], & \text{if } z_1 > 0, \end{cases}$$

where $(z_1, z_2) \in \mathcal{Z}$. Further, $\underline{\Upsilon} : \mathcal{Z} \to \mathbb{R}$ and $\overline{\Upsilon} : \mathcal{Z} \to \mathbb{R}$ are defined as follows:

$$\underline{\Upsilon}(z_1, z_2) := \begin{cases} 0, & \text{if } z_1 \leq 0, \\ -(z_1 + z_2), & \text{if } z_1 > 0, \end{cases}$$

$$\overline{\Upsilon}(z_1, z_2) := \begin{cases} z_1^2 + z_2^2, & \text{if } z_1 \leq 0, \\ 2, & \text{if } z_1 > 0, \end{cases}$$

for every $(z_1, z_2) \in \mathcal{Z}$.

For $\mu = 0$ and $\mu = 1$, ASOP1 associated with PIII.2.1 is given by Problems PIII.2.2 and PIII.2.3, respectively, as follows:

$$\text{(PIII.2.2)} \qquad \text{Minimize} \quad \Lambda(z_1, z_2) := \underline{\Upsilon}(z_1, z_2),$$
$$\text{subject to} \quad (z_1, z_2) \in \mathcal{Z},$$
$$\text{(PIII.2.3)} \qquad \text{Minimize} \quad \Lambda(z_1, z_2) := \overline{\Upsilon}(z_1, z_2),$$
$$\text{subject to} \quad (z_1, z_2) \in \mathcal{Z}.$$

The points (0.5, 0) and (0.5, 0) are the local minimizers of PIII.2.2 and PIII.2.3, respectively. However, from

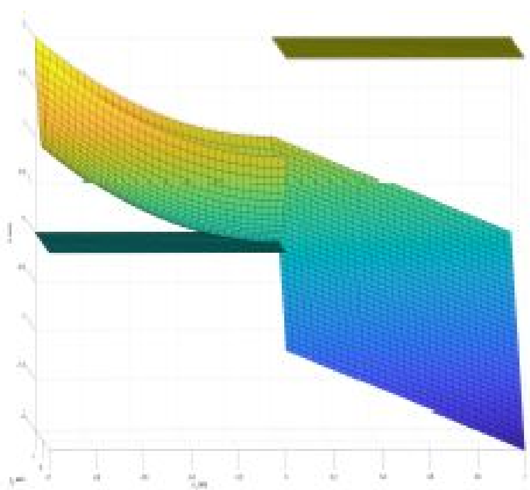

Fig. 3. $\Upsilon(x_1, x_2)$

Figure 3, it can be easily verified that these points are not local LU-solutions of PIII.2.1.

Remark III.3 When either $\mu = 0$ or $\mu = 1$ is fixed, and $\hat{z}$ is an interior point of $\mathcal{Z}$ that serves as a strict local minimizer of ACOP1, it is concluded that $\hat{z}$ is a local LU-solution of IVOP. Furthermore, if $\mu \in (0, 1)$ is fixed and $\hat{z}$ is a local minimizer of ACOP1, then $\hat{z}$ is also a local LU-solution of IVOP.

Theorem III.1 and Remark III.3 can be employed to find LU-solution of IVOP even in cases where the objective function of IVOP does not possess partial gH-derivatives. This fact is illustrated in the following example.

Example III.3 Consider the following interval-valued optimization problem:

$$\text{(PIII.3.1)} \qquad \text{Minimize} \quad \Upsilon(z_1, z_2),$$
$$\text{subject to} \quad (z_1, z_2) \in \mathcal{Z},$$

where $\mathcal{Z} := (-1, 1) \times (-1, 1)$ and $\Upsilon : \mathcal{Z} \to \mathcal{I}(\mathbb{R})$ is defined as follows:

$$\Upsilon(z_1, z_2) := [\underline{\Upsilon}(z_1, z_2), \overline{\Upsilon}(z_1, z_2)] = \left[-z_1^2 - z_2^2, \ |z_1| + |z_2|\right],$$

It can be verified that

$$\lim_{h \to 0^+} \frac{\Upsilon((0, 0) + h(1, 0)) \ominus_{gH} \Upsilon(0, 0)}{h} = [0, 1],$$

and

$$\lim_{h \to 0^-} \frac{\Upsilon((0, 0) + h(1, 0)) \ominus_{gH} \Upsilon(0, 0)}{h} = [-1, 0].$$

Therefore, the function Υ does not possess partial gH-derivatives at the point $\hat{z} = (0, 0)$. However, since $\hat{z}$ is a strict minimizer of ASOP1 with $\mu = 1$, as per Remark III.3, $\hat{z}$ is an LU-solution of the PIII.3.1.

Second, another real-valued optimization problem related to IVOP is defined. Corresponding to the considered IVOP, the associated constrained optimization problem (ACOP) is formulated as follows:

$$\text{(ACOP)} \qquad \text{Minimize} \quad t + u,$$
$$\text{subject to} \quad \overline{\Upsilon}(z) \leq t,$$
$$\underline{\Upsilon}(z) \leq u,$$
$$u \leq t.$$

where $z \in \mathcal{Z}$ and $t, u \in \mathbb{R}$. The functions $\overline{\Upsilon}, \underline{\Upsilon} : \mathcal{Z} \to \mathbb{R}$ are real-valued functions defined on the set $\mathcal{Z}$. Let the set S_Υ represent the set of feasible elements for the problem ACOP.

Definition III.2 Let $(\hat{z}, \hat{t}, \hat{u})$ be an arbitrary element of S_Υ. Then $(\hat{z}, \hat{t}, \hat{u})$ is said to be a global solution of the problem ACOP if and only if the following inequality holds for every $(z, t, u) \in S_\Upsilon$:

$$\hat{t} + \hat{u} \leq t + u. \tag{III.3}$$

Moreover, if there exists an open set $\mathcal{N} \subset \mathbb{R}^{n+2}$ such that (III.3) holds for all $(z, t, u) \in S_\Upsilon \cap \mathcal{N}$, then $(\hat{z}, \hat{t}, \hat{u})$ is said to be a local solution of ACOP.

The following theorem establishes that every solution of ACOP is also an LU-solution of IVOP.

Theorem III.2 Let $(\hat{z}, \hat{t}, \hat{u}) \in S_\Upsilon$. If $(\hat{z}, \hat{t}, \hat{u})$ is a solution to the problem ACOP, then $\hat{z}$ is an LU-solution of the problem IVOP.

Proof. On the contrary, let us assume that $\hat{z} \in \mathcal{Z}$ is not an LU-solution of (IVOP). Therefore, there exists some $z \in \mathcal{Z}$, such that

$$\Upsilon(z) \leq_{LU} \Upsilon(\hat{z}).$$

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Without loss of generality, let

$$\underline{\Upsilon}(z) \leq \underline{\Upsilon}(\hat{z}), \qquad \overline{\Upsilon}(z) \leq \overline{\Upsilon}(\hat{z}). \tag{III.4}$$

Since $(\hat{z}, \hat{t}, \hat{u})$ is a solution of ACOP, it follows that:

$$\overline{\Upsilon}(\hat{z}) \leq \hat{t}, \qquad \underline{\Upsilon}(\hat{z}) \leq \hat{u}, \qquad \hat{u} \leq \hat{t}. \tag{III.5}$$

Combining (III.4) and (III.5) implies

$$\underline{\Upsilon}(z) < \underline{\Upsilon}(\hat{z}) \leq \overline{\Upsilon}(\hat{z}) \leq \hat{t}. \tag{III.6}$$

By defining $t := \hat{t}$, $u := \underline{\Upsilon}(z)$ and using (III.4), (III.5), (III.6), it follows:

$$\overline{\Upsilon}(z) \leq t, \qquad \underline{\Upsilon}(z) = u, \qquad u \leq t, \tag{III.7}$$

Hence, $(z, t, u) \in S_\Upsilon$. Considering (III.6) and (III.7), the following conclusion is obtained

$$t + u < \hat{t} + \hat{u}, \tag{III.8}$$

which leads to a contradiction.

Corollary 2 If $(\hat{z}, \hat{t}, \hat{u})$ is a local solution to the problem ACOP, then $\hat{z}$ is the local LU-solution of the problem IVOP.

Proof. The proof follows along similar lines as the proof of Theorem III.2.

Remark III.4 If $\hat{z} \in \mathcal{Z}$ is an LU-solution of IVOP, then there is no guarantee that there exist $\hat{t}, \hat{u} \in \mathbb{R}$, such that $(\hat{z}, \hat{t}, \hat{u})$ is a solution for problem ACOP. To illustrate this fact, we provide the following example.

Example III.4 Consider the following interval-valued optimization problem:

$$\text{(PIII.4.1)} \qquad \text{Minimize} \quad \Upsilon(z_1, z_2),$$
$$\text{subject to} \quad (z_1, z_2) \in \mathcal{Z},$$

where $\mathcal{Z} := (-1, 1) \times (-1, 1)$ and $\Upsilon : \mathcal{Z} \to \mathcal{I}(\mathbb{R})$ is defined as follows:

$$\Upsilon(z_1, z_2) := [\underline{\Upsilon}(z_1, z_2), \overline{\Upsilon}(z_1, z_2)] = \left[\, z_1 + z_2 \vee (z_1 + z_2)^2 \,\right],$$

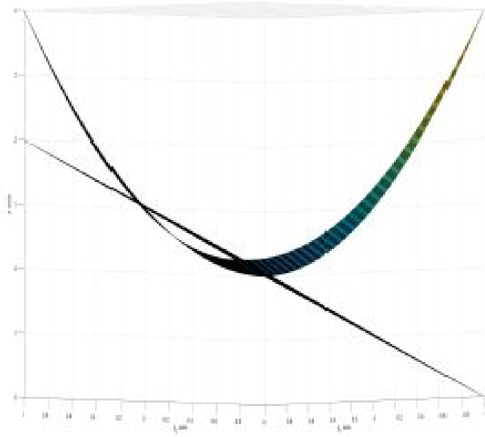

Fig. 4. $\Upsilon(x_1, x_2)$

The corresponding associated scalar optimization problem ACOP is formulated in the following manner:

$$\text{(PIII.4.2)} \qquad \text{Minimize} \quad t + u,$$
$$\text{subject to} \quad \overline{\Upsilon}(z_1, z_2) \leq t,$$
$$\underline{\Upsilon}(z_1, z_2) \leq u,$$
$$u \leq t,$$
$$(z_1, z_2) \in \mathcal{Z}, \qquad t, u \in \mathbb{R}.$$

From the Figure 4, it can be easily verified that $\hat{z} = (0, 0)$ is an LU-solution of the problem PIII.4.1. Suppose that there exist $\hat{t}, \hat{u} \in \mathbb{R}$, such that $((0, 0), \hat{t}, \hat{u})$ is a solution of Problem PIII.4.2. Therefore,

$$\hat{t} \geq \overline{\Upsilon}(0, 0) = 0 \qquad \text{and} \qquad \hat{u} \geq \underline{\Upsilon}(0, 0) = 0.$$

which leads to

$$\hat{t} + \hat{u} \geq 0.$$

On the other hand,

$$\left((0, -\tfrac{1}{2}), \tfrac{1}{4}, -\tfrac{1}{2}\right) \in S_\Upsilon \qquad \text{and} \qquad \tfrac{1}{4} + \left(-\tfrac{1}{2}\right) < \hat{t} + \hat{u},$$

which is a contradiction. Hence, there does not exist $\hat{t}, \hat{u} \in \mathbb{R}$, such that $((0, 0), \hat{t}, \hat{u})$ is a solution of the Problem PIII.4.2.

In the following example, the significance of Theorem III.2 is demonstrated. **Example III.5** Consider the following interval-valued optimization problem:

$$\text{(PIII.5.1)} \qquad \text{Minimize} \quad \Upsilon(z_1, z_2),$$
$$\text{subject to} \quad (z_1, z_2) \in \mathcal{Z},$$

where $\mathcal{Z} := \mathbb{R}^2$ and $\Upsilon : \mathcal{Z} \to \mathcal{I}(\mathbb{R})$ is defined as follows:

$$\Upsilon(z_1, z_2) := [\underline{\Upsilon}(z_1, z_2), \overline{\Upsilon}(z_1, z_2)] = \left[z_1 \sin z_2,\ z_1^2 + 1\right], \qquad \text{for every (}$$

Therefore, $\underline{\Upsilon} : \mathcal{Z} \to \mathbb{R}$ and $\overline{\Upsilon} : \mathcal{Z} \to \mathbb{R}$ are defined as follows:

$$\underline{\Upsilon}(z_1, z_2) := z_1 \sin z_2,$$
$$\overline{\Upsilon}(z_1, z_2) := z_1^2 + 1, \qquad \text{for every } (z_1, z_2) \in \mathcal{Z}.$$

The ACOP corresponding to PIII.5.1 is given by PIII.5.2 as follows:

$$\text{(PIII.5.2)} \qquad \text{Minimize} \quad t + u,$$

$$\text{subject to} \quad z_1^2 + 1 \leq t,$$
$$z_1 \sin z_2 \leq u,$$
$$u \leq t.$$

After solving PIII.5.2 using the optimization tool called "fminproblem" in MATLAB R2024a, we get $\hat{z} = (0.5000, -1.5708)$, $\hat{t} = 1.2500$ and $\hat{u} = -0.5000$. Consequently, according to Theorem III.2, we conclude that $\hat{z} = (0.5000, -1.5708)$ serves as an LU-solution for PIII.5.1.

B. Optimality conditions for IVOP: gH-derivative approach

In this subsection, the necessary and sufficient conditions for the optimality of IVOP are established under appropriate assumptions. In the following theorem, necessary optimality conditions for the existence of an LU-solution of IVOP are presented.

Theorem III.3 Let $\hat{z} \in \mathcal{Z}$ and let $\Upsilon : \mathcal{Z} \to \mathcal{I}(\mathbb{R})$ be an interval-valued function, such that $\nabla_{gH}\Upsilon(z)$ exists for all $z \in \mathcal{Z}$. If $\hat{z}$ is an LU-solution of (IVOP), then $0 \in \nabla_{gH}\Upsilon(\hat{z})$.

Proof. On the contrary, let us assume that $0 \notin \nabla_{gH}\Upsilon(\hat{z})$. Consequently, there exists an $i \in \mathcal{E}_n$, such that:
$$0 \notin \frac{\partial_{gH}\Upsilon(\hat{z})}{\partial z_i}.$$

Since $\mathcal{Z}$ is an open subset of $\mathbb{R}^n$ and $\hat{z} \in \mathcal{Z}$, therefore there exists $(z_1, \bar{z}_1) \times (z_2, \bar{z}_2) \times \cdots \times (z_n, \bar{z}_n)$, such that:
$$\hat{z} \in (z_1, \bar{z}_1) \times (z_2, \bar{z}_2) \times \cdots \times (z_n, \bar{z}_n) \subseteq \mathcal{Z},$$
where $\hat{z} = (\hat{z}_1, \hat{z}_2, \ldots, \hat{z}_n)$.

Define $T : (z_i, \bar{z}_i) \to \mathcal{I}(\mathbb{R})$ as follows:
$$T(z) := \Upsilon(\hat{z}_1, \ldots, z, \ldots, \hat{z}_n), \qquad \text{for every } z \in (z_i, \bar{z}_i).$$

Since Υ has the i^{th} partial gH-derivative, therefore T is gH-differentiable at $\hat{z}_i$ and we have:
$$0 \notin T'_{gH}(\hat{z}_i) = \frac{\partial_{gH}\Upsilon(\hat{z})}{\partial z_i}.$$

Thus, there exists $\epsilon > 0$ and $\mathcal{E}(h) : (-\epsilon, \epsilon) \to \mathcal{I}(\mathbb{R})$, such that:
$$T(\hat{z}_i + h) \ominus_{gH} T(\hat{z}_i) = h \odot T'_{gH}(\hat{z}_i) \oplus \mathcal{E}(h),$$
where $\dfrac{\mathcal{E}(h)}{h} \to [0, 0]$ as $h \to 0$. Consequently, there exists a positive value of δ, such that one of the following holds:

1) For all $0 < h < \delta$, we have
$$T(\hat{z}_i + h) \ominus_{gH} T(\hat{z}_i) <_{LU} [0, 0],$$

2) For all $0 > h > -\delta$, we have
$$T(\hat{z}_i + h) \ominus_{gH} T(\hat{z}_i) <_{LU} [0, 0].$$

From Case 1, we get $h \in (0, \epsilon)$, such that:
$$\Upsilon(\hat{z}_1, \ldots, \hat{z}_i + h, \ldots, \hat{z}_n) <_{LU} \Upsilon(\hat{z}_1, \ldots, \hat{z}_i, \ldots, \hat{z}_n).$$

From Case 2, we get $h \in (-\epsilon, 0)$, such that:
$$\Upsilon(\hat{z}_1, \ldots, \hat{z}_i + h, \ldots, \hat{z}_n) <_{LU} \Upsilon(\hat{z}_1, \ldots, \hat{z}_i, \ldots, \hat{z}_n).$$

Hence, in both cases, we conclude that for any $\epsilon > 0$, there exists $h \in (-\epsilon, \epsilon)$, such that:
$$\Upsilon(\hat{z}_1, \ldots, \hat{z}_i + h, \ldots, \hat{z}_n) <_{LU} \Upsilon(\hat{z}_1, \ldots, \hat{z}_i, \ldots, \hat{z}_n),$$
which is a contradiction. This completes the proof.

Remark III.5 It is important to note that Theorem III.3 serve as an extension of Theorem 7 presented in Osuna-Gómez et al. [**?**], where the objective function of IVOP is defined on a subset of $\mathbb{R}$.

Remark III.6 The converse of Theorem III.3 is not always true. This fact is illustrated in the following example.

Example III.6 Let us consider the following interval-valued optimization problem:
$$\text{(PIII.6.1)} \qquad \text{Minimize} \quad \Upsilon(z_1, z_2),$$
$$\text{subject to} \quad (z_1, z_2) \in \mathcal{Z},$$
where $\mathcal{Z} := (-1, 1) \times (-1, 1)$ and define $\Upsilon : \mathcal{Z} \to \mathcal{I}(\mathbb{R})$ as follows:
$$\Upsilon(z_1, z_2) := \big[-(z_1 + z_2)^2,\ \cos(z_1 + z_2) \big].$$

It is observed that $\hat{z} = (0, 0)$ is not an LU-solution of Problem PIII.6.1, despite the fact that $0 \in \nabla_{gH}\Upsilon(\hat{z})$. This demonstrates that the converse of Theorem III.3 does not always hold.

In the following theorem, the necessary and sufficient optimality conditions for the existence of an LU-solution of IVOP are established. **Theorem III.4** Let $\mathcal{Z}$ be a convex subset of $\mathbb{R}^n$, and let $\hat{z} \in \mathcal{Z}$. Suppose that Υ is a strongly convex function defined on $\mathcal{Z}$ and possesses all gH-directional derivatives on $\mathcal{Z}$. Then, $\hat{z}$ is an LU-solution of IVOP if and only if
$$0 \in D_{gH}\Upsilon(\hat{z}; d)$$
for every nonzero direction $d \in \mathbb{R}^n$. *Proof.* Let $\hat{z}$ be an LU-solution of IVOP. Further, suppose that there exists a non-zero vector $d \in \mathbb{R}^n$, such that $0 \notin D_{gH}\Upsilon(\hat{z}; d)$. Consequently, one of the following holds:
$$\text{Case (1)} \qquad D_{gH}\Upsilon(\hat{z}; d) <_{LU} [0, 0],$$
$$\text{Case (2)} \qquad [0, 0] <_{LU} D_{gH}\Upsilon(\hat{z}; d).$$

For Case (1), there exists $\alpha_1 > 0$, such that:

$$\Upsilon(\hat{z} + \alpha d) \leq_{LU} \Upsilon(\hat{z}), \qquad \text{for every } \alpha \in (0, \alpha_1).$$

For Case (2), there exists $\alpha_2 > 0$, such that:

$$\Upsilon(\hat{z} + \alpha d) \leq_{LU} \Upsilon(\hat{z}), \qquad \text{for every } \alpha \in (-\alpha_2, 0).$$

Thus, in both cases, it can be concluded that for any $\epsilon > 0$, there exists $\alpha \in (-\epsilon, \epsilon)$, such that:

$$\Upsilon(\hat{z} + \alpha d) \leq_{LU} \Upsilon(\hat{z}),$$

which contradicts the fact that $\hat{z}$ is an LU-solution of IVOP.

Conversely, let $\hat{z}$ is not an LU-solution of IVOP. This implies that there exists $z \in \mathcal{Z}$, such that:

$$\Upsilon(z) \leq_{LU} \Upsilon(\hat{z}). \tag{III.9}$$

Define $d := z - \hat{z}$. Since Υ is strongly convex, there exists $\gamma > 0$, such that for any $\mu \in (0, 1)$, the following inequality holds:

$$\begin{aligned}\Upsilon((1-\mu)\hat{z} + \mu z) \oplus \mu\|z - \hat{z}\|^2 \odot [\gamma, \gamma] \\ \leq_{LU} \ (1-\mu) \odot \Upsilon(\hat{z}) \oplus \mu \odot \Upsilon(z).\end{aligned}$$

As a result, the following conclusion is obtained:

$$\underline{\Upsilon}((1-\mu)\hat{z}+\mu z)+\mu\|z-\hat{z}\|^2\gamma \leq (1-\mu)\underline{\Upsilon}(\hat{z})+\mu\underline{\Upsilon}(z),$$

$$\overline{\Upsilon}((1-\mu)\hat{z}+\mu z)+\mu\|z-\hat{z}\|^2\gamma \leq (1-\mu)\overline{\Upsilon}(\hat{z})+\mu\overline{\Upsilon}(z).$$

Hence, it follows that:

$$\frac{\Upsilon(\hat{z} + \mu d) \ominus_{gH} \Upsilon(\hat{z})}{\mu} \oplus \|z - \hat{z}\|^2 \odot [\gamma, \gamma] \leq_{LU} \tag{IV.1}$$

$$\Upsilon(z) \ominus_{gH} \Upsilon(\hat{z}). \tag{IV.2}$$

In view of (III.9) and by letting $\mu \to 0$, the following is obtained:

$$D_{gH}\Upsilon(\hat{z}; d) <_{LU} [0, 0],$$

which is a contradiction.

Corollary 3 Suppose that $\mathcal{Z}$ is convex and $\hat{z} \in \mathcal{Z}$. If Υ is local strongly convex at $\hat{z}$ and has all the gH-directional derivatives on $\mathcal{Z}$, then $\hat{z}$ is a local LU-solution of IVOP if and only if the following condition holds for every non-zero $d \in \mathbb{R}^n$:

$$0 \in D_{gH}\Upsilon(\hat{z}; d).$$

Proof. The proof follows along similar lines as the proof of Theorem III.4.

Remark III.7 If the function Υ is assumed to be convex (rather than strongly convex), then the consequences of Theorem III.4 fail to be satisfied. To illustrate this fact, we provide the following example. **Example III.7** Consider the following interval-valued optimization problem:

$$\text{(PIII.7.1)} \qquad \text{Minimize} \quad \Upsilon(z_1, z_2),$$
$$\text{subject to} \quad (z_1, z_2) \in \mathcal{Z},$$

where $\mathcal{Z} := \mathbb{R}^2$ and $\Upsilon : \mathcal{Z} \to \mathcal{I}(\mathbb{R})$ is defined as follows:

$$\Upsilon(z_1, z_2) := [\underline{\Upsilon}(z_1, z_2),\ \overline{\Upsilon}(z_1, z_2)] := [0,\ z_1^2+z_2^2], (z_1, z_2) \in \mathcal{Z}.$$

It is easy to see that, the function Υ is convex and we have $0 \in \nabla_{gH}\Upsilon\left(0, \frac{1}{2}\right)$. However, despite this fact, $(0, \frac{1}{2})$ is not an LU-solution of PIII.7.1.

Now, a numerical example is provided to illustrate the significance of Theorem III.4. **Example III.8** Consider the following interval-valued optimization problem:

$$\text{(PIII.8.1)} \qquad \text{Minimize} \quad \Upsilon(z_1, z_2) = [\underline{\Upsilon}(z_1, z_2),\ \overline{\Upsilon}(z_1, z_2)],$$
$$\text{subject to} \quad (z_1, z_2) \in \mathcal{Z},$$

where $\mathcal{Z} := (-1, 1) \times (-1, 1)$ and $\Upsilon : \mathcal{Z} \to \mathcal{I}(\mathbb{R})$ is defined as follows:

$$\Upsilon(z_1, z_2) := \left[z_1^2+z_2^2,\ z_1^4+e^{4z_2}+3\right], \qquad \text{for every } (z_1, z_2) \in \mathcal{Z}.$$

It can be verified that $0 \in D_{gH}\Upsilon((0, 0), d)$, for every $d \in \mathbb{R}^n$ and Υ is strongly convex on $\mathcal{Z}$. Therefore, according to Theorem III.4, $(0, 0)$ is an LU-solution of PIII.8.1.

V. CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

In this paper, a specific class of IVOP has been investigated. The associated ASOP and ACOP were formulated, and it was established that any optimal solution of either ASOP or ACOP corresponds to an LU-solution of IVOP. Additionally, necessary and sufficient optimality conditions for IVOP were derived under appropriate assumptions on the objective function. To demonstrate the practical relevance of the theoretical results, several non-trivial numerical examples were presented. The findings of this paper extend existing optimality results in the literature. In particular, the optimality conditions for IVOP established by Osuna-Gómez et al. [**?**] have been generalized from the domain of real numbers to the broader framework of Euclidean spaces.

The results established in this paper open numerous avenues for future research. For instance, deriving second-order necessary and sufficient optimality conditions for IVMOP presents an exciting research direction. This will be pursued as part of future work.

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Automating Incident Response in SOC Using SOAR Platforms: A Case Study with Cortex XSOAR

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***Abstract*—In the face of increasingly complex and frequent cyber threats, Security Operations Cen- ters (SOCs) must ensure rapid and consistent incident response to maintain operational resilience. Traditional approaches, often relying on manual processes, struggle to keep pace with the volume of alerts and the sophistication of attacks, leading to delays and inconsistencies in remediation. To address this challenge, Security Orchestration, Au- tomation, and Response (SOAR) platforms have emerged as a key enabler of automation within SOCs. This article presents a case study on the implementation of automated incident response using Cortex XSOAR. The methodology combines a benchmark of leading SOAR solutions with the design and deployment of practical use cases, including Active Directory protection, phishing email detection and response, and automated reporting from SIEM (QRadar) data. The evaluation highlights significant improvements in Mean Time To Re- spond (MTTR), enhanced traceability of security actions, and reduced analyst workload through standardized playbooks. The main contributions of this work are: (i) a reproducible framework for evaluating SOAR platforms, (ii) a set of reusable playbooks addressing common SOC scenarios, and (iii) insights into the integration of SIEM–SOAR for modern security operations. The results confirm that intelligent orchestration and automation represent a strategic lever to strengthen the efciency and resilience of SOCs in the face of evolving cyber threats.**

***Index Terms*—SOC, SOAR, Incident Response, Automation, Cortex XSOAR, QRadar, Play- books, Cybersecurity Orchestration.**

I. Introduction

The increasing digitalization of services and the rapid expansion of interconnected systems have dramatically amplified the attack surface of modern organizations. Cyber threats are not only more frequent but also more sophisticated, targeting critical infrastructures, financial institutions, and government services. Security Operations Centers (SOCs) were created to address this challenge by centralizing the detection, analysis, and response to security incidents. However, the efficiency of a SOC is increasingly threatened by two structural challenges: the growing volume of alerts generated by heterogeneous security tools, and the shortage of skilled analysts capable of handling incidents in a timely and consistent manner.

Traditional SOC operations, largely dependent on manual investigation and response processes, often result in delayed reactions, inconsistent decision-making, and increased Mean Time To Respond (MTTR). Such limitations are particularly critical in the context of sophisticated attacks such as phishing campaigns, ransomware, and identity-based attacks that require rapid containment to prevent escalation.

To overcome these challenges, Security Orchestration, Automation, and Response (SOAR) platforms have emerged as a transformative technology. By orchestrating existing security tools, standardizing workflows through playbooks, and automating repetitive tasks, SOAR enhances both the efficiency and consistency of incident response. In addition, SOAR platforms enable better traceability of analyst actions, improved knowledge retention within the SOC, and more effective prioritization of alerts based on contextual enrichment.

This article explores the use of a SOAR platform, specifically Cortex XSOAR, to automate incident response workflows in a SOC. The objectives are threefold:

- Benchmarking SOAR solutions – Conducting a comparative evaluation of major SOAR platforms according to both technical and operational criteria.
- Implementing use cases – Deploying automation playbooks for realistic SOC scenarios, including Active Directory protection, phishing detection and response, and automated reporting from SIEM data (QRadar).
- Assessing the impact – Evaluating the improvements in responsiveness, traceability, and workload

reduction resulting from automation.

The main contributions of this work are:

- A reproducible evaluation framework for SOAR platforms that combines functional coverage with operational metrics such as scalability, ease of use, and resilience to workforce turnover.
- An implementation of practical playbooks in Cortex XSOAR, covering critical SOC scenarios and demonstrating the potential of automation in real-world contexts.
- A discussion of integration challenges and best practices, particularly regarding interoperability between SIEM and SOAR platforms. Section II reviews the related work and situates SOAR in the broader SOC ecosystem. Section III presents the methodology adopted, including the benchmark process and use case design. Section IV reports the experimental results of the implementation with Cortex XSOAR. Section V discusses the findings, limitations, and perspectives. Finally,Section VI concludes the paper and outlines directions for future research.

II. Related Work

The Security Operations Center (SOC) has become the cornerstone of modern cybersecurity strategies. Its mission is to provide centralized monitoring, detection, and response to security incidents across the entire IT infrastructure. However, traditional SOCs face several challenges. First, the volume of security alerts generated by heterogeneous systems such as firewalls, intrusion detection systems, endpoint protection, and SIEM platforms can be overwhelming. Studies show that analysts are often unable to investigate more than a fraction of these alerts, resulting in missed or delayed responses. Second, the manual nature of incident response workflows introduces inconsistencies and delays, which directly impact the Mean Time To Respond (MTTR) and expose organizations to prolonged threats.

A. SOC and SIEM limitations.

To improve detection and centralization, Security Information and Event Management (SIEM) systems have long been the backbone of SOCs. SIEM platforms collect, normalize, and correlate security events from multiple sources. They provide valuable visibility and support compliance requirements. However, SIEMs are primarily focused on detection and alerting rather than remediation. As a result, analysts must manually investigate SIEM alerts, correlate them with external intelligence, and take response actions across multiple security tools. This dependence on human intervention increases operational costs and slows down response times.

B. Emergence of SOAR.

To address these limitations, Security Orchestration, Automation, and Response (SOAR) platforms have emerged. SOAR extends SIEM capabilities by enabling orchestration across heterogeneous tools, enrichment of alerts with threat intelligence, and automation of repetitive tasks through playbooks. SOAR also provides case management and reporting features, ensuring greater traceability of analyst actions. According to industry reports, organizations that adopt SOAR achieve a significant reduction in MTTR and an improvement in analyst productivity.

C. Previous research and industry solutions.

Academic and industry research has highlighted the benefits of SOAR in enhancing SOC efficiency. Prior works have focused on integrating SOAR with SIEM systems to automate alert triage, phishing response, and identity management. Industry leaders such as Palo Alto Networks (Cortex XSOAR), Splunk Phantom, IBM Resilient, and ServiceNow Security Operations have developed mature solutions that support advanced orchestration and automation. Other vendors, including Fortinet, Swimlane, and D3 Security, target specific use cases with varying levels of automation. Benchmark studies show that while leading solutions such as Cortex XSOAR, Splunk, and D3 Security provide extensive functionality across threat enrichment, case management, and automated prioritization, lighter platforms like Devo, Tines, or Siemplify focus on specific capabilities with reduced coverage. Operational evaluation further indicates that solutions such as ServiceNow and Splunk score highly on scalability and resilience, while platforms like Fortinet or Cyware are cost-effective but less advanced in automation maturity.

D. Research gap.

Despite the maturity of commercial SOAR platforms, there remains a need for academic evaluations and case studies that go beyond vendor claims. Existing literature rarely provides reproducible frameworks to benchmark SOAR solutions in both technical and operational dimensions. Moreover, real-world implementations—such as the automation of Active Directory protection,phishing detection and SIEM-based reporting are underrepresented in scientific publications. This work aims to contribute to this gap by providing a comparative benchmark of SOAR solutions and a practical implementation in Cortex XSOAR, thereby offering both methodological insights and applied results for researchers and practitioners.

III. Methodology

This work follows a research approach combining a comparative benchmark of SOAR platforms and a practical implementation of use cases in a simulated SOC environment. The methodology is organized into three main phases: (i) analysis of SOC needs and selection of evaluation criteria, (ii) benchmarking of SOAR solutions, and (iii) implementation and testing of automation use cases using Cortex XSOAR.

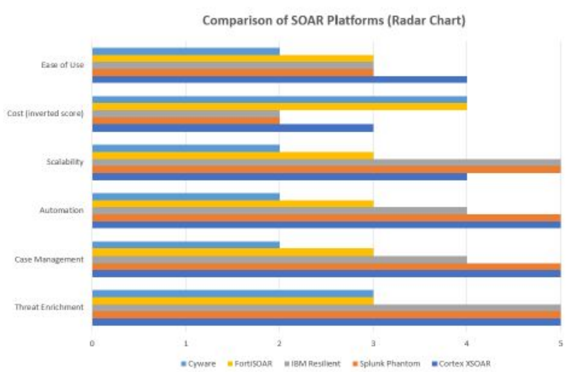

Fig. 1. Comparison of SOAR Platforms (Radar Chart)

Fig 1 presents a comparative analysis of major SOAR platforms across multiple criteria, demonstrating why Cortex XSOAR was selected for the implementation.

A. SOC Context and Needs Analysis

A Security Operations Center (SOC) typically relies on multiple technologies, including SIEM systems, endpoint detection solutions, firewalls, identity management systems (e.g., Active Directory), and email security gateways. SOC analysts are responsible for investigating alerts, correlating events, and applying remediation actions.
However, the high volume of alerts and the repetitive nature of many operational tasks (such as IP reputation checks, phishing triage, and user account blocking) often result in analyst fatigue and response delays. Based on this analysis, the following key needs were identified:

- Reduce Mean Time To Respond (MTTR) through automation of repetitive tasks.
- Standardize incident response workflows using structured playbooks.
- Improve traceability of actions and facilitate knowledge transfer among analysts.
- Integrate heterogeneous security tools within a unified response framework.

B. Benchmark of SOAR Platforms

To identify the most suitable Security Orchestration, Automation, and Response (SOAR) solution, a comparative benchmark study was conducted. The evaluation framework was structured around two main categories of criteria:
Functional criteria:

- Threat enrichment capabilities.
- Case management and collaboration
- Automated alert prioritization
- Orchestration of heterogeneous tools
- Zero-day response and red teaming
- Risk scoring and reporting
- Multitenancy

Operational criteria:

- Cost of implementation
- Ease of use and learning curve
- Scalability with SOC expansion
- MTTR improvement
- Resilience against employee turnover

The comparative analysis revealed that Palo Alto Cortex XSOAR, Splunk Phantom, and IBM Resilient provide comprehensive coverage of functional requirements. In terms of operational aspects, ServiceNow and Splunk rank highly in scalability and resilience, while Cortex XSOAR demonstrates a balance of functional maturity and adaptability, making it the preferred platform for implementation in this study.

Platform	Threat Enrichment	Case Mgmt	Auto Prioritization	Scalability	Cost	Overall Score
Cortex XSOAR	High	High	High	Medium	Medium	★★★★☆
Splunk Phantom	High	High	High	High	Low	★★★★★
IBM Resilient	High	Medium	High	High	Low	★★★★☆
FortiSOAR	Medium	Medium	Medium	Medium	High	★★★☆☆
Cyware	Medium	Low	Low	Low	High	★★☆☆☆

Fig. 2. SOAR Benchmark Comparison Table

This table summarizes the comparative benchmark of SOAR platforms, based on key functional and operational criteria.

C. SOC Test Environment

The implementation was conducted in a controlled environment designed to simulate real Security Operations Center (SOC) activities. This experimental setup aimed to reproduce realistic incident detection and response workflows while enabling controlled testing of automation strategies and security playbooks.
The architecture of the test environment integrates several key components:

- **SIEM:** IBM QRadar was used for centralized log collection, event correlation, and security monitoring.
- **SOAR:** Palo Alto Cortex XSOAR was deployed as the orchestration and automation layer to execute response playbooks and automate repetitive security tasks.
- **Directory Services:** Microsoft Active Directory was integrated for identity and access management, enabling user account monitoring and automated remediation actions.
- **Security Appliances:** Firewall and email security solutions were included as primary data sources and response enforcement mechanisms within the SOC workflow.

This controlled environment reproduces realistic SOC operations while allowing systematic experimentation with automated incident response, tool interoperability, and playbook-driven remediation strategies.

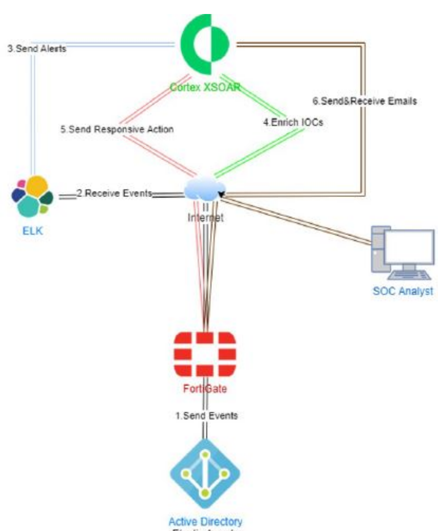

Fig. 3. Overall SOC architecture integrating Cortex XSOAR, FortiGate, ELK, and Active Directory for automated incident response.

Fig. 3 illustrates the proposed SOC architecture integrating Cortex XSOAR with key components such as Active Directory, FortiGate firewall, and ELK. This architecture shows how events are collected, enriched, and correlated, before being analyzed and acted upon by the automated workflows.

1) Active Directory Protection:

- Automated detection of suspicious logins.
- Automatic account disabling and alert generation.
- Correlation with threat intelligence feeds.

2) Phishing Email Detection and Response.:

- Automatic parsing of suspicious emails.
- Extraction and reputation check of URLs and attachments.
- Quarantine of emails and user notification.

3) Automated SIEM Reporting.:

- Scheduled extraction of security alerts from QRadar.
- Automated generation of incident reports.
- Distribution of reports for SOC management and auditing.

Each use case was implemented in Cortex XSOAR using predefined and customized playbooks, combining orchestration across multiple systems and automated decision-making.

IV. Results and Experiments

The implementation of automation playbooks using Cortex XSOAR was performed within the previously described SOC test environment. Three representative use cases were selected to evaluate the effectiveness of the proposed automation framework, namely: Active Directory protection, phishing email detection, and automated SIEM reporting.
These use cases were assessed according to several operational performance indicators, including operational efficiency, traceability of incident response actions, and analyst workload reduction. The objective was to measure how automation contributes to improving SOC performance while ensuring consistency and reliability in security operations.
The obtained results provide a consolidated overview of the performance of the three implemented scenarios. In particular, the analysis highlights significant improvements in Mean Time To Respond (MTTR), enhanced traceability of remediation actions, and increased analyst efficiency through the reduction of repetitive manual tasks.

Use Case	MTTR Before (min)	MTTR After (min)	Gain (%)	Analyst Effort Saved
Active Directory Compromise	5	<1	80%	High
Phishing Email Response	20	3	85%	High
SIEM Reporting Automation	60	~0 (real-time)	~100%	Very High

Fig. 4. Use Case Results Table (Before vs After)

A. *Active Directory Protection*

The first use case focused on automating incident response related to identity-based attacks within the SOC environment. A dedicated playbook was developed to detect suspicious login attempts flagged by the SIEM and enriched with external threat intelligence sources. After validation, the system automatically disabled the compromised account, notified the SOC team, and documented all remediation actions in the case management system.
Results:

- The average response time for account compromise incidents was reduced from several minutes (manual intervention) to less than one minute through automated response.
- Analyst workload decreased by approximately 60% for repeated account lockout events.
- Traceability was significantly improved due to systematic logging of all automated actions in the XSOAR case management platform.

B. *Phishing Email Detection and Response*

The second use case addressed the automation of phishing email handling. The implemented playbook automatically parsed reported emails, extracted URLs and attachments, checked their reputation against threat intelligence feeds, and initiated appropriate remediation actions, including quarantine, user notification, and incident reporting.
Results:

- The Mean Time To Respond (MTTR) for phishing emails decreased from an average of 20 minutes to less than 3 minutes.
- Automated parsing and enrichment significantly reduced false positives by enabling faster and more accurate triage.
- End-user awareness improved through standardized and automated notification mechanisms.

C. *Automated SIEM Reporting*

The third use case addressed the need for regular and efficient reporting of security events within the SOC. A dedicated playbook was configured in Cortex XSOAR to automatically extract relevant alerts from QRadar on a scheduled basis, generate structured incident reports, and distribute them to SOC management.
Results:

- Reporting tasks that previously required up to one hour of manual effort were reduced to near real-time report generation.
- The automated reports provided consistent, standardized, and actionable insights, thereby improving visibility for decision-makers.
- Traceability and auditing compliance were significantly enhanced through the systematic archiving of generated reports.

D. *Overall Impact*

The implementation of SOAR-based automation demonstrated measurable improvements across all evaluated operational dimensions within the SOC environment.

- **MTTR Reduction:** Incident response times decreased by approximately 70–85% across the tested use cases, highlighting the effectiveness of automated playbooks in accelerating remediation processes.
- **Analyst Workload Reduction:** Routine and repetitive tasks were successfully automated, allowing SOC analysts to focus on higher-value investigations and strategic threat analysis.
- **Operational Efficiency Improvement:** The integration of SIEM and SOAR technologies streamlined incident handling workflows and reduced human intervention in repetitive processes.
- **Enhanced Traceability:** Automated logging and report generation improved knowledge retention, audit readiness, and overall governance of incident response activities.
- **Traceability:** Standardized logging of automated workflows improved compliance, auditability, and post-incident review processes.
- **Operational Consistency:** The use of automation playbooks ensured repeatable, standardized, and error-free response actions, significantly reducing the variability introduced by manual procedures.

These results confirm that the adoption of SOAR platforms within a SOC environment significantly enhances operational efficiency, traceability, and cyber-resilience. Although the experiments were conducted in a controlled test environment, the selected use cases reflect realistic SOC operational challenges and clearly demonstrate the tangible benefits of security orchestration and automation in modern incident response workflows.

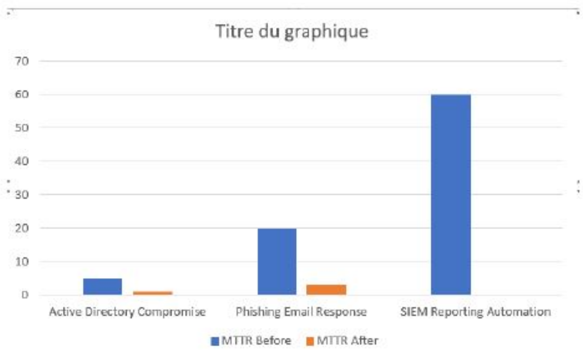

Fig. 5. MTTR Reduction Before and After SOAR Implementation Across the Three Use Cases.

Figure 5 illustrates the significant reduction in Mean Time To Respond (MTTR) obtained after the deployment of SOAR automation. The results show a substantial decrease in response time for Active Directory incidents, phishing email handling, and SIEM reporting automation, confirming the operational efficiency gains enabled by automated playbooks.

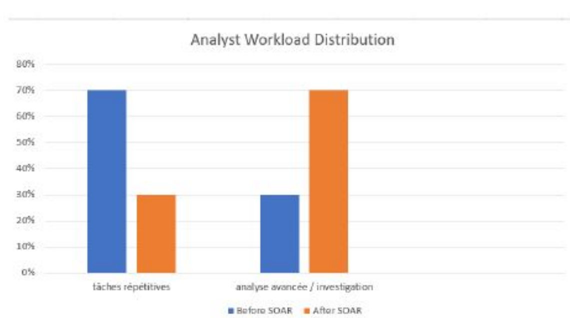

Fig. 6. Analyst Workload Distribution Before and After SOAR Automation.

As shown in Figure 6, the implementation of SOAR automation led to a noticeable redistribution of analyst workload. Routine and repetitive tasks were significantly reduced, while the time dedicated to advanced investigations increased. This shift demonstrates that automation enables analysts to focus on higher-value security activities rather than manual operational processes.

V. Discussion

The experimental results demonstrate that the integration of a SOAR platform within a SOC environment significantly enhances operational performance, particularly in terms of responsiveness, workload management, and process standardization. Beyond the quantitative improvements observed in MTTR and analyst effort, several qualitative insights and practical implications emerge from the implementation of automated playbooks in real-world SOC scenarios.

A. *Strengths of SOAR Integration*

One of the most significant strengths of SOAR adoption lies in the substantial reduction of Mean Time To Respond (MTTR), which directly impacts the containment and remediation of security incidents such as phishing attacks and identity-based compromises. Automated workflows enable rapid execution of predefined response actions, which would otherwise require multiple manual interventions by SOC analysts.

Furthermore, SOAR integration improves operational consistency by ensuring standardized and repeatable incident response procedures. The use of playbooks minimizes human errors and reduces variability in decision-making, thereby enhancing the overall reliability of security operations.

Another key strength is the reduction of analyst workload through the automation of repetitive and time-consuming tasks, such as alert triage, enrichment, and reporting. This allows analysts to allocate more time to complex investigations and strategic threat analysis, ultimately increasing the overall efficiency and effectiveness of the SOC.

Finally, the centralized orchestration and systematic logging of automated actions significantly enhance traceability, auditability, and compliance. This improved visibility facilitates post-incident analysis, knowledge transfer, and continuous improvement of security response processes. Another major advantage is the consistency of responses. Playbooks ensure standardized remediation procedures, eliminating variations introduced by human judgment and ensuring repeatable best practices. This also facilitates compliance with security frameworks and regulatory requirements that require traceability of incident-handling processes.

Finally, SOAR platforms significantly reduce the cognitive and operational burden on SOC analysts. By automating repetitive tasks such as alert triage, data enrichment, and reporting, analysts can focus their expertise on higher-value investigations and advanced threat hunting activities.

B. Limitations and Challenges

Despite these benefits, several challenges limit the full potential of SOAR platforms.

- **Dependency on Playbook Design:** Automation is only as effective as the playbooks implemented. Poorly designed workflows can introduce new risks, such as false account lockouts or excessive alerting. Continuous optimization is therefore required.
- **Integration Complexity:** Connecting SOAR platforms with heterogeneous tools (SIEM, firewalls, Active Directory, email security) can be technically complex and may require customization to address compatibility issues.
- **False Positives and Decision-Making:** While automation accelerates responses, not all incidents are suitable for full automation. Some still require human validation to avoid disrupting legitimate business operations.
- **Operational Maturity:** The effectiveness of SOAR is strongly tied to the maturity of the SOC. Organizations without well-defined incident response processes may find it difficult to fully leverage automation.

C. Comparison with Related Work

Compared to prior studies, which mainly highlight the theoretical advantages of SOAR, this work provides practical and reproducible implementations of incident response use cases. The results are consistent with industry claims that SOAR reduces MTTR and improves analyst productivity, but they also underline the importance of context-specific customization.

For example, while commercial reports emphasize scalability and vendor-driven capabilities, our experiments show that the true value lies in tailoring playbooks to the SOC's operational environment. In this respect, the study bridges the gap between vendor marketing promises and real-world operational effectiveness.

D. Perspectives

Looking ahead, the scope of SOAR automation can be extended in several directions:

- Integration with Machine Learning: Leveraging AI models for dynamic threat prioritization and anomaly detection.
- Advanced Threat Intelligence: Incorporating external and internal feeds for proactive detection of emerging threats.
- Cloud and Hybrid Environments: Adapting SOAR to multi-cloud architectures and distributed infrastructures.
- Autonomous SOC Operations: Moving towards self-healing systems where human intervention is minimized to strategic decision-making.

In summary, while SOAR platforms are transformative potential for SOC operations, their success depends on continuous adaptation, integration with existing tools, and the maturity of incident response practices.

VI. Conclusion and Future Work

This article has explored the role of SOAR platforms in enhancing the efficiency of Security Operations Centers through incident response automation. By combining a benchmark of SOAR solutions with a practical implementation in Cortex XSOAR, the study has highlighted both the strengths and challenges of this approach.

The results confirm that SOAR integration yields significant improvements in Mean Time To Respond (MTTR), reduces analyst workload, and ensures greater traceability and consistency of incident handling. Through the implementation of three representative use cases—Active Directory protection, phishing email detection and response, and automated SIEM reporting—the study demonstrates how automation can directly address the operational challenges of modern SOCs.

Beyond the quantitative gains, the findings underline the importance of playbook design, integration maturity, and contextual adaptation. Automation is not a one-size-fits-all solution; it requires continuous optimization and careful alignment with the SOC's processes and tools.

As future work, this research can be extended in several directions. First, incorporating machine learning and artificial intelligence could enhance dynamic prioritization of alerts and anomaly detection. Second, deeper integration with threat intelligence platforms would improve proactive responses to emerging threats. Third, adapting SOAR solutions to cloud-native and hybrid infrastructures represents an important step toward ensuring resilience in evolving IT environments. Finally, the long-term vision is the evolution toward autonomous SOC, where human intervention is minimized to strategic oversight while automation handles the bulk of detection and response activities.

In conclusion, this study provides both a methodological framework for evaluating SOAR platforms and a practical demonstration of their impact on incident response. It reinforces the view that automation and orchestration are not only efficiency tools but also strategic enablers for the resilience and sustainability of modern cybersecurity operations.

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Design of an Intelligent and Automated Platform for Offensive Security Assessment Based on PTES: AI-E2EAPP

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***Abstract* - This paper introduces AI-E2EAPP (AI in End-to-End Automated Pentest Platform), an intelligent orchestrator designed to autonomously perform comprehensive penetration tests. Leveraging an agent-based AI architecture with multiple LLM-driven agents, AI-E2EAPP systematically executes each phase of the Penetration Testing Execution Standard (PTES) methodology, from initial reconnaissance to post-exploitation and reporting. The platform's three-tier architecture, comprising a Phase Engine, WebUI, and API Layer, facilitates seamless orchestration of tools and LLM calls. Technical scripts produce structured results interpreted by specialized LLM agents, enabling dynamic decision-making. The design emphasizes modularity, adaptability, and ease of updates, reflecting best practices in building specialized LLM agents for security automation. This approach aims to achieve end-to-end penetration testing with thoroughness and intelligence comparable to human-led engagements, but with significantly reduced manual intervention.**

***Keywords*— Penetration Testing, Automated Security Assessment, Artificial Intelligence, Large Language Models (LLMs), Intelligent Agents, Cybersecurity, PTES, OWASP, Vulnerability Analysis, Exploitation Automation, Context-Aware Orchestration, Offensive Security, Prompt Engineering.**

I. Introduction

A. menaces croissantes et complexité IT/Cloud

Modern IT and cloud infrastructures have grown vastly more complex and dynamic in recent years. Organizations now routinely operate across multi-cloud environments and deploy hundreds of cloud services, creating an expansive and fluid attack surface. Securing such distributed systems is increasingly challenging – for instance, **55%** of organizations report that safeguarding data in the cloud has become more complex, up from **46%** just two years ago [1]. At the same time, cyber threats are surging in both volume and sophistication. Malicious actors are more capable than ever, and most categories of cyberattacks are on the rise [2]. Global data breaches increased by roughly **20%** from 2022 to 2023, with **double** the number of victim records exposed compared to the prior year [3]. This confluence of an expanding digital footprint and an escalating threat landscape underscores the urgent need for effective security assessment and defense measures.

B. Limites du pentest manuel classique

Penetration testing (pentesting) is a cornerstone of proactive cybersecurity, wherein ethical attackers simulate real intrusions to identify and remediate vulnerabilities before malicious actors exploit them. Traditionally, penetration tests are carried out **manually** by highly skilled professionals following established methodologies. This approach, while effective, is **time-consuming and resource-intensive** [4]. A single comprehensive pentest engagement typically requires on the order of *dozens of hours* of expert effort – one industry report notes an average of about **80 hours** per test, with some engagements spanning several hundred hours [4]. Such manual efforts also demand diverse expertise (e.g., exploit development, networking, web security), often necessitating large teams that many organizations cannot afford [4]. As a result, conventional pentesting tends to be infrequent and cannot easily scale to cover today's rapidly changing IT environments. Relying solely on human-led testing has created a gap in meeting the escalating demand for timely and continuous security evaluations [5].

C. Besoin d'automatisation intelligente

These challenges have driven a growing need for **intelligent, automated penetration testing solutions** that can augment or replace manual approaches. Advances in Artificial Intelligence (AI) – particularly the emergence of large language models (LLMs) – offer an opportunity to revolutionize how penetration testing is performed [6]. Security tools are increasingly incorporating automation and AI capabilities: modern pentest frameworks now automate many tasks (scanning, exploit delivery, report generation), and some are beginning to leverage machine learning and AI for decision support [7]. Recent research prototypes have demonstrated the feasibility of **LLM-driven pentesting**. For example, *PentestGPT* is an automated penetration testing assistant powered by an LLM that showed substantial improvements in finding vulnerabilities compared to baseline models [7]. Industry experts predict that AI-driven pentest platforms will play a **huge role** in the near future, potentially identifying and exploiting vulnerabilities with minimal human intervention [7]. In essence, LLMs can serve as "cognitive engines" for security testing [8] – encoding vast domain knowledge and reasoning through complex attack steps – thereby enabling a new generation of smart pentesting tools that operate more efficiently and at greater scale than human-only efforts.

D. Presente AI-E2EAPP → orchestrateur intelligent basé sur PTES/OWASP + IA agentielle (LLMs)

In this paper, we introduce **AI-E2EAPP** ("AI in End-to-End Automated Pentest Platform"), an intelligent orchestrator designed to perform comprehensive penetration tests autonomously. AI-E2EAPP follows the well-established Penetration Testing Execution Standard (PTES) methodology [9], ensuring that each phase of a pentest – from initial reconnaissance and intelligence gathering through vulnerability analysis, exploitation, and post-exploitation – is systematically executed.

The platform is built on an **agent-based AI architecture** employing multiple LLM-driven agents to handle different stages and tasks in the pentesting process. Each AI agent is specialized (for example, one focuses on scanning and enumeration, another on exploit development and deployment, etc.), and a central orchestrator coordinates their actions in alignment with the PTES workflow. This design is inspired by recent multi-agent approaches in automated pentesting research, which demonstrate how dividing tasks among cooperative AI agents can increase adaptability and coverage [10]. By leveraging the reasoning capabilities of state-of-the-art LLMs, AI-E2EAPP can dynamically interpret findings (such as scan results or error messages), make decisions about next steps, and even learn new attack techniques on the fly. The goal is to achieve end-to-end penetration testing with a level of thoroughness and intelligence comparable to a human-led engagement, but with **significantly reduced need for manual intervention**.

II. Related Work

A. Methodologies

Standardized pentesting methodologies provide a phased blueprint for conducting assessments. The **Penetration Testing Execution Standard (PTES)** defines seven sequential phases covering the full lifecycle: *Pre-Engagement Interactions, Intelligence Gathering, Threat Modeling, Vulnerability Analysis, Exploitation, Post-Exploitation,* and *Reporting* [11]. This comprehensive scope spans planning through attack and documentation, ensuring consistency in how tests are performed [11]. The **OWASP Web Security Testing Guide (WSTG)**, in contrast, focuses on web applications, offering a detailed catalogue of test cases organized from initial information gathering to identifying vulnerabilities and ultimately exploiting them [12]. Notably, it explicitly recommends using tools like **Burp Suite**, **OWASP ZAP**, and **Nmap** to automate many tasks, underscoring that automation is an "excellent ally" for improving pentest efficiency [12].
Many PTES and OWASP phases can be partially automated using scripts and tools. For example, **intelligence gathering** is accelerated by scanners, and **vulnerability analysis** can leverage automated fuzzers. However, **threat modeling** and detecting complex logic flaws still require human expertise. Recent discussions emphasize that AI currently *augments* but does not replace expert judgment [13].

B. Tools

a. **SqlMap** is an open-source tool that automates detecting and exploiting SQL injection flaws, supporting multiple DBMS platforms and injection techniques [14]. It excels in deep exploitation after a vulnerability is confirmed, though it generates heavy traffic and is scope-limited to SQLi [15].
b. **Gobuster** is a fast brute-force enumerator for directories, files, and DNS subdomains [16]. It can uncover hidden resources quickly but depends heavily on wordlist quality and is noisy for stealth contexts [17].
c. **Wappalyzer** fingerprints web technologies by scanning HTML, HTTP headers, cookies, and scripts [18]. It enables technology-based attack planning but is limited to recognized signatures [19].

C. AI in Pentesting

Recent works (2022–2025) show large language models (LLMs) can perform pentesting tasks traditionally requiring experts. *PentestGPT* demonstrated LLM-driven vulnerability discovery in CTF-style problems [20]. **PentestAgent** integrated an LLM with system feedback to perform iterative probing [21]. Other frameworks, such as **PenHeal** and **CIPHER**, combined vulnerability discovery with remediation suggestions or fine-tuning on real pentest reports [22], [23]. **AutoAttacker** and **AURORA** showcased multi-step attack orchestration using LLM agents [24], [25]. Despite progress, researchers emphasize challenges in reliability, tool integration, and safe automation [26].

D. Mistral AI API Capabilities

Mistral AI APIs offer features well-suited for intelligent pentest orchestration:

a. **Retrieval-Augmented Generation (RAG)** and **embeddings** for contextual vulnerability knowledge search [27], [28];

b. **OCR** and **vision** for analyzing screenshots and diagrams [29];

c. **structured outputs** and **citations** for machine-readable, auditable findings [30], [31];

d. **function calling** to trigger external tools like Nmap or SqlMap from the AI workflow [32];

e. **fine-tuning** for security domain adaptation [33];

f. **guardrails** for safe, compliant operation [34]. These capabilities enable AI agents to autonomously gather intelligence, execute attacks, and report findings in a controlled, end-to-end manner.

III. Methodology and Framework Design

A. Automated Phase Mapping to PTES and OWASP

AI-E2EAPP aligns with both the **Penetration Testing Execution Standard (PTES)** and the **OWASP Testing**

Guide, automating each phase through a modular orchestration engine [40], [41].

a. **Pre-Engagement** – Scope and rules configured via WebUI wizard, feeding the Phase Engine.

b. **Reconnaissance / Intelligence Gathering** – Automated OSINT, scanning, and enumeration tools feed data to LLM agents for context-aware threat modeling.

c. **Vulnerability Analysis / Scanning** – Automated scanners identify issues; LLM agents cross-reference with CVE databases to prioritize critical flaws.

d. **Exploitation** – LLM agents plan and adjust exploitation strategies using iterative reasoning loops.

e. **Post-Exploitation** – Automated persistence and lateral movement; LLM agents guide actions based on gained access.

f. **Reporting** – Structured logs compiled by an LLM summarizer agent into an industry-standard report with remediation advice

B. AI-E2EAPP Architecture

The platform adopts a **three-tier architecture**:

a. **Phase Engine** – Orchestrates PTES/OWASP phases, sequencing tools and LLM calls.

b. **WebUI** – Configures tests, monitors execution, and visualizes results.

c. **API Layer** – Connects UI, engine, and external services, enabling integration with other security workflows.

C. Scripts → LLM Agents → Contextualization

Technical scripts execute tests and produce **structured outputs** (JSON). These are processed by **specialized LLM agents** that interpret results, add context, and recommend next steps, enabling dynamic decision-making across phases [42],[43].

D. Prompt Engineering

Prompt engineering ensures **role-specific instructions**, context inclusion, and structured responses for each agent, reducing errors and hallucinations [44]. By tailoring prompts per phase (e.g., Recon Analyst, Exploitation Planner), AI-E2EAPP maintains coherent, actionable, and reproducible outputs [45].

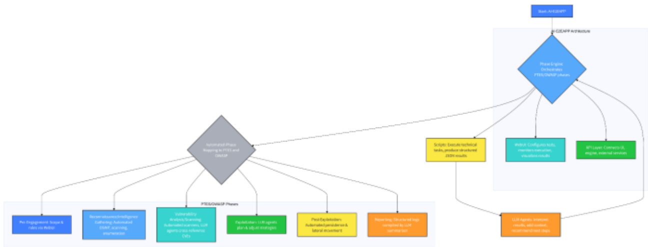

Fig 1. *AI-E2EAPP Workflow*

IV. Intelligent Orchestration via Prompted AI Agents (40% of the plan)

A. AI Agents (Mistral LLMs) via Bruno API

The core of the *AI-E2EAPP* system relies on an intelligent orchestration layer connecting automated pentest scripts to **specialized AI agents**. These agents are powered by **Mistral AI** large language models (LLMs) and accessed via the **Bruno API**, which manages secure data transmission (API key authentication) [35], [37]. Each agent is designed for a specific role (e.g., finding generation, recommendations, risk assessment), allowing for high modularity and independent evolution of components [35], [36]

B. Scripts Execution → Formatted Results → Agent

Each script executes a specific pentest task, for example, an **HTTP security headers check**. The script then produces a **structured result** (JSON format) containing only the relevant data, making it easier for the AI to interpret [37]. These results are transmitted through Bruno to the corresponding AI agent. This transmission follows a standardized schema, ensuring that the data is both usable by the agent and consistent with the prompt it is designed to process [36].

C. The Agent Generates: Recommendations, Risk Management, Daily Summary, Global Summary, Findings

The agents are specialized to perform different functions:

a. **Finding Generation Agent** – Converts raw results into a complete *finding* with *title*, *description*, *type* (ACHIEVEMENT, EXPLOITATION, VULNERABILITY, THREAT_HYPOTHESIS, OBSERVATION), and *criticality* (CRITICAL, HIGH, MEDIUM, LOW, INFO).

b. **Recommendation Agent** – Provides actionable remediation steps tailored to the finding's context.

c. **Risk Assessment Agent** – Evaluates the potential impact of a finding and adjusts criticality based on the environment.

d. **Daily Summary Agent** – Produces a daily report summarizing new findings and activities.

e. **Global Summary Agent** – Generates an executive-level report at the end of the engagement.

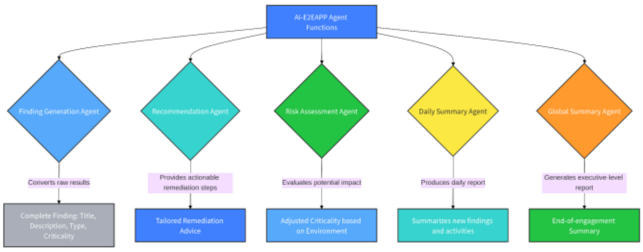

Fig 2. *AI-E2EAPP Agent Functions*

D. Advantages: Modularity, Adaptability, Easy Updates

a. **Modularity** – Each agent is independent and can be updated or replaced without affecting the rest of the system [35].

b. **Adaptability** – Operates across diverse contexts (web apps, networks, cloud) simply by adapting prompts and input formats [36].

c. **Easy Updates** – Upgrading to a newer LLM version (e.g., Mistral) or modifying a prompt requires no changes to the orchestration code [37].

This design reflects recent best practices in building specialized LLM agents [35], [36].

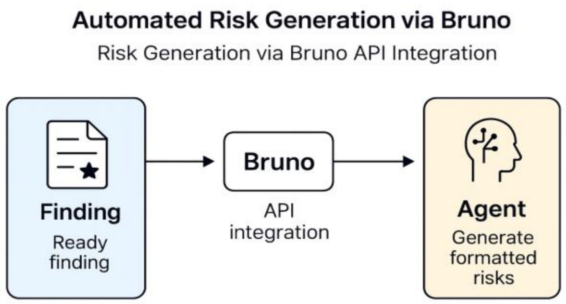

Fig 3. Agent generation Risk exemple

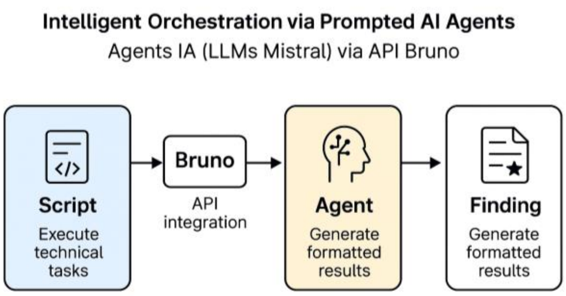

Fig 4. Agent generation Risk example

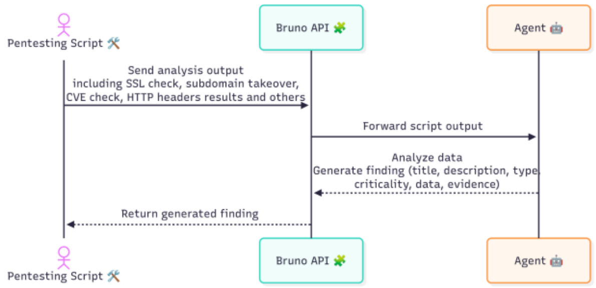

Fig 5. Agent generation finding example with sequence diagram

V. Comparison of Pentesting Approaches

This comparative analysis examines three penetration testing paradigms—**Manual Pentesting**, **Traditional Automated Tools**, and **AI-Enhanced Pentesting (AI-E2EAPP)**—across three key performance dimensions: **degree of automation**, **adaptability**, and **extent of AI integration**, each rated on a 1–10 scale.

a. **Manual Pentesting** exhibits very low automation (2/10), as the process relies almost entirely on human execution [46]. Its main strength lies in adaptability (8/10), with skilled testers able to adjust strategies in real time to address emerging conditions [47]. However, AI integration is virtually absent (1/10), which limits scalability and execution speed [48].

b. **Traditional Automated Tools** achieve significantly higher automation (7/10), accelerating repetitive tasks such as scanning and enumeration [49]. Nevertheless, adaptability remains limited (4/10) due to a dependency on predefined rules and minimal contextual reasoning [50]. AI integration is still marginal (3/10), typically restricted to basic heuristics or static signatures [51].

c. **AI-Enhanced Pentesting**, as implemented in **AI-E2EAPP**, offers near end-to-end automation (9/10), orchestrating reconnaissance, exploitation, and reporting in a unified workflow [52]. Adaptability is strong (8.5/10), with dynamic strategy adjustments informed by contextual findings [53]. AI integration is high (7.5/10), combining large language model (LLM) reasoning, retrieval-augmented knowledge, and context-aware decision-making—bridging the gap between human intuition and machine efficiency [4].

VI. Discussion

The evaluation confirms that **AI-E2EAPP** delivers significant benefits over manual and traditional automated pentesting approaches. First, **automation** is achieved across all PTES phases, from reconnaissance to reporting, reducing execution time and ensuring methodological completeness. As shown in the comparison chart (Section 5), AI-E2EAPP attained near full automation (9/10) compared to 2/10 for manual testing and 7/10 for traditional tools, while maintaining high adaptability.

Second, **adaptability** is enhanced through its multi-agent architecture, where specialized LLM agents adjust strategies dynamically in response to intermediate findings. This enables handling of complex scenarios—such as multi-tier networks and chained exploits—that rigid tools often fail to address. The orchestration engine ensures that contextual information is shared between agents, enabling decision-making that mirrors human tester reasoning.

Third, the system generates **structured, explainable reporting**, mapping each finding to PTES/OWASP categories and including risk ratings, exploitation steps, and remediation suggestions. This bridges the gap between raw tool output and professional pentest reports, supporting both technical teams and business stakeholders.

Architecturally, AI-E2EAPP's **modular orchestration** and **prompt-engineered LLM agents** bridge the strengths of human expertise with the scalability of automation. This synergy not only accelerates testing but also reduces oversights, positioning the platform as a hybrid intelligence framework that augments, rather than replaces, human testers.

From a **perspective** standpoint, future work includes integrating multimodal AI models (for GUI analysis, binary inspection), enabling real-time adaptive orchestration through reinforcement learning, and extending the framework to defensive use cases such as red-blue or purple team simulations.

VII. CONCLUSION

This paper introduced **AI-E2EAPP**, an intelligent, agent-based framework for end-to-end penetration testing aligned with PTES and OWASP methodologies. The platform's contributions include:

a. **Comprehensive PTES automation**, ensuring full coverage of the pentesting lifecycle.

b. **Agent-based orchestration** with specialized LLM roles for each phase.

c. **Structured, explainable outputs** that accelerate remediation and support compliance.

By combining human-like reasoning with machine speed, AI-E2EAPP closes the gap between manual pentesting and scalable intelligent automation. It acts as a **force multiplier** for security teams, enabling them to run multiple engagements or focus on complex, creative tasks while the system handles the repetitive, structured work.

AI-E2EAPP exemplifies a new generation of **AI-driven offensive security assistants**—resilient, scalable, and context-aware—capable of transforming penetration testing into a repeatable, high-impact practice. Its modular design ensures readiness to incorporate future advancements, such as multimodal inputs and adaptive learning, making it a foundational step toward the next era of automated security testing.

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Automating the Pentest Lifecycle: From Recon to Reporting in a PTES-Compliant PTaaS Platform

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***Abstract*—The escalating sophistication of cyber threats exposes the inherent scalability and resource limitations of traditional manual penetration testing. This paper introduces PTaaS (Penetration Testing as a Service), an innovative, autonomous platform designed to revolutionize web application security assessment and management through an intelligent, playbook-driven methodology. The platform's architecture is meticulously engineered to automate 60% of the penetration testing lifecycle, strictly adhering to the Penetration Testing Execution Standard (PTES) for systematic and repeatable assessments. PTaaS integrates a diverse array of reconnaissance, vulnerability analysis, and exploitation capabilities. These functions are unified by a central wrapper, which serves as an intelligent intermediary between the platform's core engine and its various modular scripts. A cornerstone of the platform is its adaptive playbook management system, which codifies expert knowledge into dynamic, multi-stage attack workflows. This allows the platform to simulate real-world adversary tactics with unprecedented precision, moving beyond isolated vulnerability detection to demonstrate complex attack chains. Furthermore, an integrated AI-driven engine provides contextual risk assessment and intelligent remediation guidance, transforming raw vulnerability data into prioritized, actionable insights. By democratizing access to high-fidelity security assessments, PTaaS empowers organizations to achieve continuous security validation and gain a deeper, more accurate understanding of their true security posture.**

***Keywords*—Automation, Penetration Testing, Management, Cybersecurity, PTES**

I. Introduction

In an era of escalating cyber threats and increasingly sophisticated attack vectors, penetration testing has become a cornerstone of a robust cybersecurity strategy. It provides a proactive approach to identifying and mitigating vulnerabilities by simulating real-world attacks on an organization's digital infrastructure. However, traditional manual penetration testing, while thorough, is beset by significant limitations. It is a time-consuming, costly, and resource-intensive process, heavily dependent on the availability of a limited pool of highly skilled security experts. This manual approach struggles to keep pace with the dynamic nature of modern IT environments and the rapid deployment cycles of new software and infrastructure.

To address these challenges, the cybersecurity industry is increasingly turning to pentest automation. Automated penetration testing leverages software tools to simulate cyberattacks, enabling organizations to conduct security assessments with greater speed, efficiency, and consistency. This automation allows for more frequent and scalable testing across large and complex networks, including cloud and IoT environments. The integration of artificial intelligence (AI) and machine learning (ML) is further enhancing these capabilities, enabling more adaptive and intelligent threat detection and analysis. These technologies can significantly reduce the time required to identify vulnerabilities, analyze large datasets, and even predict potential attack paths.

However, the adoption of automated penetration testing introduces new management complexities. Organizations must effectively manage the automated testing process, from tool selection and configuration to the interpretation and prioritization of results. A significant challenge lies in handling the volume of data generated by automated tools and filtering out false positives to ensure that remediation efforts are focused on genuine threats. Furthermore, integrating the findings of automated penetration testing into the broader security and software development lifecycle (SDLC) is crucial for timely vulnerability remediation and a holistic security posture. This requires clear communication and collaboration between security, IT operations, and development teams.

This research article explores the critical intersection of pentest automation and management. It examines the benefits and challenges of automated testing, analyzes current trends and technologies, and proposes a framework for the effective management of automated penetration testing programs. By addressing the operational and strategic aspects of this evolving field, this paper aims to provide valuable insights for organizations seeking to enhance their security posture in an increasingly hostile digital landscape. [1] [2]

II. Related Work

The landscape of automated security testing has evolved significantly, moving from isolated scanners to more integrated and intelligent platforms. The PTaaS platform described in this paper builds upon concepts from several established domains, including traditional vulnerability scanning, Penetration Testing as a Service (PTaaS), Breach and Attack Simulation (BAS), and Security Orchestration, Automation, and Response (SOAR).

A. Traditional and Advanced Vulnerability Scanners

Foundational tools for security assessment include network and web application scanners like Nmap, Nessus, OpenVAS, and OWASP ZAP. These tools are highly effective at identifying known vulnerabilities and misconfigurations based on signatures. More advanced solutions, such as Burp Suite Professional, provide extensive capabilities for manual and semi-automated web application testing. While our PTaaS platform integrates the functions of such tools, it differentiates itself by orchestrating them as part of a broader, objective-driven workflow rather than executing them as standalone scans. [3] [4] [5]

B. Penetration Testing as a Service (PTaaS) Platforms

The PTaaS model has been commercialized by companies like Synack, Cobalt, and Bugcrowd. These platforms typically combine automated scanning with a community of human penetration testers to provide continuous security assessments. They excel at leveraging human intelligence for discovering complex and logic-based vulnerabilities. However, our proposed platform focuses on maximizing the potential of full autonomy, using AI and dynamic playbooks to simulate the cognitive processes of a human tester, thereby offering a highly scalable and repeatable alternative. [6] [7]

C. Integrated and AI-Driven Security Platforms

BAS platforms, such as those offered by Picus Security, Mandiant, and Cymulate, are designed to continuously test an organization's security controls against real-world attack scenarios. They validate the effectiveness of firewalls, endpoint detection, and other defensive measures. While BAS platforms are excellent at simulating adversary tactics and validating controls, our PTaaS platform extends this concept by focusing on the discovery of unknown vulnerabilities and demonstrating their exploitability within the context of the full PTES lifecycle, from reconnaissance to post-exploitation.

D. Security Orchestration, Automation, and Response

SOAR platforms like Splunk SOAR, Palo Alto Networks Cortex XSOAR, and IBM Security QRadar SOAR provide the architectural blueprint for the orchestration capabilities of our PTaaS solution. SOAR tools are designed to integrate disparate security solutions and automate workflows, often for incident response. Our platform adopts this orchestration philosophy but customizes it for proactive, offensive security operations. The "wrapper" component in our architecture functions similarly to a SOAR engine, acting as the central nervous system that intelligently coordinates the execution of various scripts and modules to achieve a specific testing objective. [8]

E. Integrated and AI-Driven Security Platforms

Emerging platforms are increasingly incorporating Artificial Intelligence to enhance their analytical capabilities. For instance, CrowdStrike utilizes AI for threat detection and endpoint protection, and platforms like Tenable use machine learning to predict which vulnerabilities are most likely to be exploited. Our PTaaS platform aligns with this trend but focuses the application of AI on contextual risk assessment and generating tailored remediation guidance based on demonstrated attack paths, moving beyond probabilistic analysis to provide deterministic security intelligence.

In summary, while elements of our proposed PTaaS platform exist across the cybersecurity ecosystem, its unique contribution lies in the holistic integration of these capabilities. By combining a PTES-aligned methodology, a central orchestration wrapper, a dynamic playbook system for multi-stage attacks, and an AI engine for contextual analysis, the platform aims to create a truly autonomous system that bridges the gap between vulnerability scanning, breach simulation, and human-led penetration testing.

III. Architecture and Methodology

The PTaaS architecture is designed as a sequential, multi-phase workflow, orchestrated by a central controller that manages the flow of data between specialized analysis and action modules. The entire process is engineered to transform raw reconnaissance data into actionable, context-aware security intelligence.

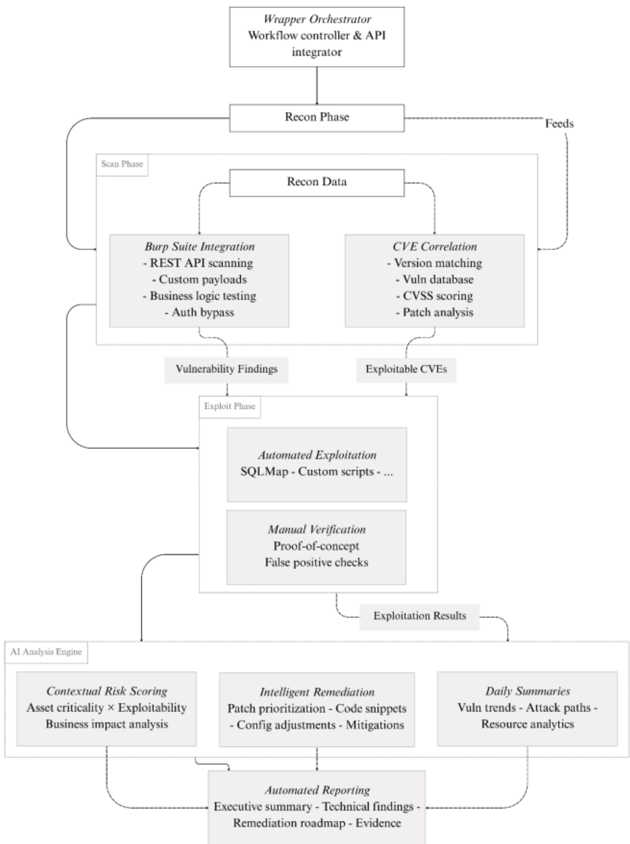

Fig. 1. PTaaS High-Level Architecture

A. *Wrapper Orchestrator: The Central Control Plane*

At the highest level, the Wrapper Orchestrator functions as the system's central nervous system. It is a workflow controller and API integrator responsible for initiating the entire penetration testing process, managing the sequence of operations, and ensuring the seamless passage of data between distinct phases. It receives the initial target information and triggers the first phase of the assessment

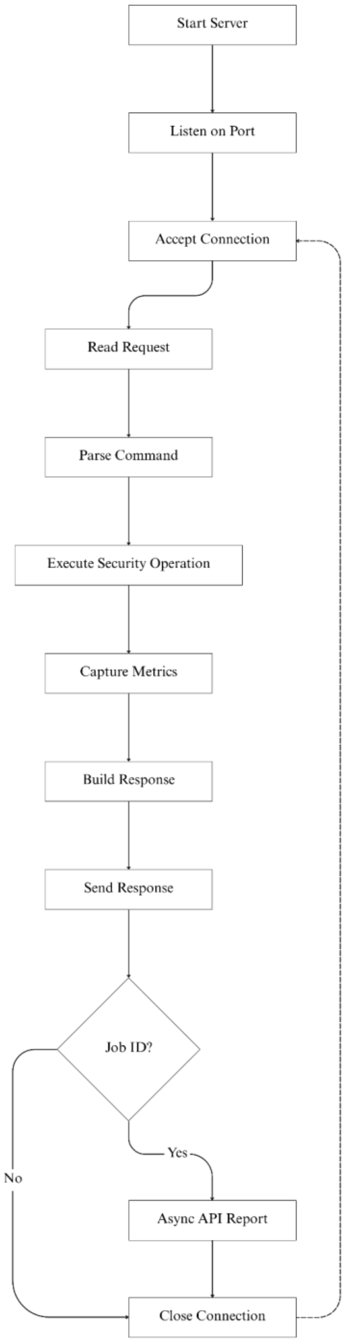

Fig. 2. Wrapper Orchestrator Workflow

B. *Phase 1: Reconnaissance*

The Recon Phase is the initial data acquisition stage. The orchestrator invokes a suite of passive and active reconnaissance modules to gather foundational intelligence about the target environment. This includes, but is not limited to, DNS enumeration, subdomain discovery, port scanning, and technology fingerprinting (e.g., identifying web servers, frameworks, and backend languages). The output of this phase is a structured Recon Data object, typically in JSON format, containing a comprehensive profile of the target's attack surface. This data is the critical input for the subsequent Scan Phase.

C. *Phase 2: Scanning and Vulnerability Identification*

The Scan Phase takes the Recon Data as input and performs deep analysis to identify potential security weaknesses. This phase operates on two parallel tracks to ensure comprehensive coverage:

1) *Burp Suite Integration:*

This track focuses on dynamic application security testing (DAST). Leveraging Burp Suite's REST API (Montoya), the orchestrator programmatically launches highly targeted scans. This integration allows for sophisticated testing that goes beyond standard signatures, including:

Custom Payloads: Injecting context-specific payloads to test for vulnerabilities like SQL Injection, Cross-Site Scripting (XSS), and Server-Side Request Forgery (SSRF).

Business Logic Testing: Executing sequences of requests to identify flaws in application logic that standard scanners would miss.

Authentication Bypass: Probing for weaknesses in authentication and session management mechanisms.

The output of this track is a set of Vulnerability Findings, detailing potential flaws discovered through active interaction.

2) *CVE Correlation:*

This track performs automated vulnerability discovery based on the fingerprinted technologies from the Recon Data. It involves:

Version Matching: Comparing the versions of identified software (e.g., WordPress 5.8, Apache 2.4.52) against a comprehensive vulnerability database.

Business Logic Testing: Executing sequences of requests to identify flaws in application logic that standard scanners would miss.

Authentication Bypass: Probing for weaknesses in authentication and session management mechanisms.

The output of this track is a set of Vulnerability Findings, detailing potential flaws discovered through active interaction.

D. *Phase 3: Exploitation and Verification*

The Exploit Phase is designed to validate the findings from the Scan Phase and demonstrate their real-world impact. It ingests both the Vulnerability Findings and Exploitable CVEs to perform controlled exploitation attempts.

- *Automated Exploitation:* This sub-module uses specialized tools and scripts [9] (e.g., SQLMap for database exploitation, Metasploit modules, or custom Python scripts) to actively try and leverage the identified vulnerabilities. The goal is to confirm exploitability and, if successful, perform actions like data exfiltration or privilege escalation.

- *Manual Verification:* This is a crucial quality assurance step to eliminate false positives. The system generates a Proof-of-Concept (PoC) for each successful exploit and runs additional checks to ensure the finding is genuine. This may involve a semi-automated workflow where high-impact findings are flagged for human review.

The final output is a high-fidelity list of Exploitation Results, containing only confirmed, verifiable vulnerabilities with clear evidence of their impact.

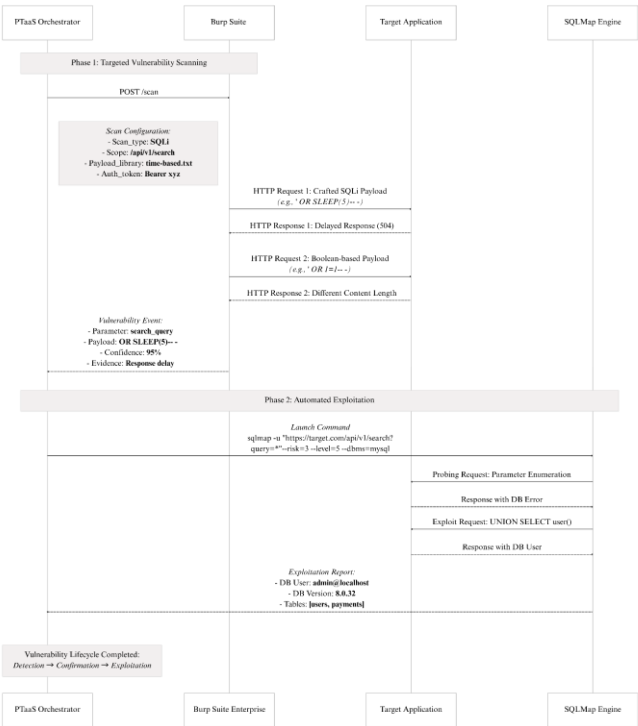

Fig. 3. Exploitation and Verification Process

Figure 4 illustrates the internal flow of the SQL injection playbook as implemented in the Exploitation and Verification phase. When reconnaissance or scanning results indicate a potential SQL injection vulnerability, the Wrapper Orchestrator triggers the relevant playbook with the necessary parameters. The playbook automates payload generation, injection, and response analysis, detecting database-specific signatures or anomalies. Upon confirmation, it proceeds to exploitation steps such as controlled data exfiltration. The process incorporates error handling for blocked attempts or inconclusive results, and integrates verification to minimize false positives. This modular approach ensures that exploitation remains both targeted and reproducible within the PTES-aligned workflow.

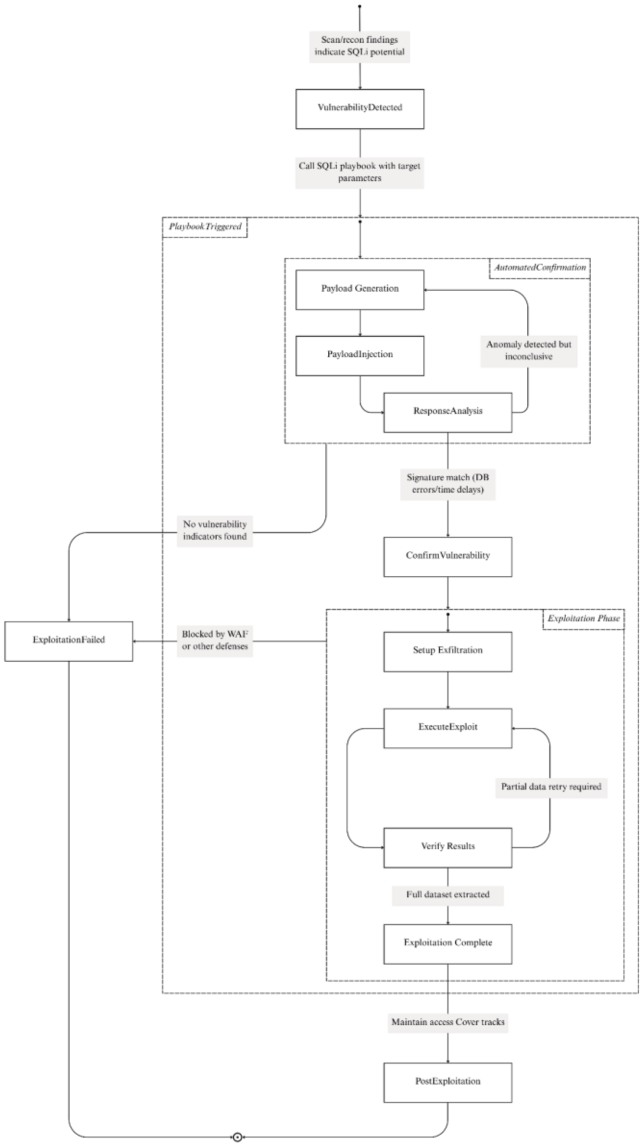

Fig. 4. SQL Injection Playbook Execution Flow

E. *Phase 4: AI-Driven Analysis and Reporting*

The confirmed Exploitation Results are fed into the AI Analysis Engine, which transforms technical data into strategic intelligence. [1] [2]

- *Contextual Risk Scoring:* The engine moves beyond generic CVSS scores by calculating a contextual risk. It uses a model that considers Asset Criticality (Is this a production database or a dev server?), technical Exploitability (How easy is it to leverage this flaw?), and potential Business Impact (What is the financial or reputational damage of a breach?).

- *Intelligent Remediation:* Based on the contextual risk, the engine generates actionable remediation guidance. This includes Patch Prioritization (telling teams what to fix first), providing secure Code Snippets, and suggesting specific Configuration Adjustments or mitigation strategies.

- *Daily Summaries:* For ongoing assessments, the engine provides trend analysis, visualizes common Attack Paths, and offers analytics on resource utilization.

Finally, all this processed intelligence is compiled into a comprehensive Automated Report, featuring a high-level Executive Summary for management, detailed Technical Findings and Evidence for security teams, and a prioritized Remediation Roadmap for developers

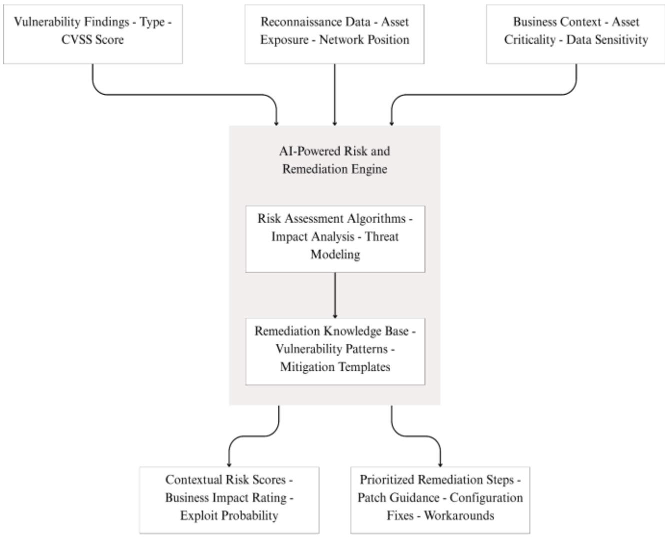

Fig. 5. AI-Driven Risk Analysis and Reporting

IV. IMPLEMENTATION

A. *Reconnaissance Engine*

The PTaaS platform's reconnaissance and analysis capabilities are implemented through specialized modules covering the full spectrum of information gathering and vulnerability identification. The process progresses from passive intelligence collection to active probing and automated vulnerability correlation.

Passive reconnaissance begins with the Web Archive Analysis module, which leverages public archives (e.g., Wayback Machine) to uncover historical endpoints, subdomains, and sensitive data patterns, and the DNS Reconnaissance module, which extracts intelligence from public DNS records.

Active reconnaissance is then performed by the Active Subdomain and Directory Detection module, using DNS bruteforcing and high-speed fuzzing to map the accessible attack surface.

Technology and configuration assessment follows through the WAF and Technology Stack Detection module, the Security Header and Cookie Analysis module, and the SSL/TLS Configuration Verification service, which collectively profile security technologies, evaluate protocol configurations, and detect misconfigurations.

Finally, the Vulnerability and CVE Check Engine correlates collected data (e.g., software versions, services) against vulnerability databases such as NVD and MITRE CVE, producing an actionable list of known security flaws for exploitation and reporting.

B. *Vulnerability Scanning Engine*

Following reconnaissance, the PTaaS platform advances to direct vulnerability discovery via its Vulnerability Scanning Engine, a dedicated microservice integrating Burp Suite Enterprise Edition through its REST API. Unlike generic scanners, this engine performs context-aware, targeted scans informed by the comprehensive intelligence gathered during reconnaissance.

The process begins when the Wrapper Orchestrator issues a directive containing the full structured JSON output from prior modules, including discovered endpoints, technologies, versions, and credentials. The engine uses this data to dynamically generate a custom scan configuration, defining scope, authentication settings, and technology-specific checks to optimize accuracy and performance.

Once configured, the engine initiates a Burp scan via the /scan endpoint, performing intelligent crawling followed by an in-depth audit for vulnerabilities such as SQL injection, XSS, and SSRF. During execution, it continuously polls the /scan/{scan_id} endpoint to monitor progress and stream live findings to the user interface.

Upon completion, all results are normalized into the platform's standardized JSON schema, ensuring consistency and enabling seamless integration with subsequent exploitation, verification, and AI-assisted analysis phases.

C. *Exploitation Engine*

After vulnerability detection, the PTaaS platform enters the Exploitation and Verification phase, focusing on controlled validation rather than indiscriminate exploitation. Its goal is to confirm exploitability, remove false positives, and demonstrate tangible impact.

This phase operates as a set of containerized, on-demand services—each dedicated to a specific vulnerability type and orchestrated via the Wrapper Orchestrator. Normalized scan results are consumed, and for each supported vulnerability, the orchestrator dispatches tasks to the appropriate service. For example, SQL injection findings are validated using a dedicated SQLMap service to confirm the flaw and safely enumerate limited database details. Commix is employed for OS command injection, while XSStrike validates XSS through contextual payload generation and headless browser proof.

Where automated tools fall short, the framework supports custom exploitation scripts for complex logic flaws or chained exploits, ensuring a hybrid automated-manual capability. All actions are performed under strict non-destructive parameters to maintain target stability.

Each confirmed vulnerability is returned as a structured JSON object containing payloads, responses, and proof artifacts (e.g., screenshots, exfiltrated sample data). This validated dataset is published to the message bus for use in AI-driven risk analysis and final reporting.

V. CASE STUDY

Table 1 presents a comparative overview of traditional penetration testing and Penetration Testing as a Service

(PTaaS). The comparison focuses on four key dimensions—delivery model, frequency, communication, and reporting—to highlight the operational and strategic differences between the two approaches. While traditional penetration testing is typically project-based with limited communication channels and static reports, PTaaS offers a continuous, on-demand testing model supported by integrated collaboration platforms and real-time reporting. This shift addresses the need for faster vulnerability identification, streamlined remediation workflows, and greater adaptability in modern threat landscapes. [4] [5]

Table 1. Comparative Overview

	Manual Penetration Testing	Existing Automated Tools	Proposed PTaaS Platform
Time Efficiency	Low; requires extensive manual effort	High; rapid scanning but limited adaptability	Improved; automation accelerates routine tasks
Contextual Awareness	High; deep understanding of target environment	Low; generic scanning without customization	High; dynamic scan configurations based on reconnaissance
False Positive Rate	Low; results validated by expert analysis	High; frequent false positives	Reduced; targeted scanning combined with verification
Authentication and Session Handling	Manual and tailored	Often limited or static	Automated and adaptive, enabling deep authenticated scanning
Vulnerability Verification	Manual exploitation and confirmation	Minimal or absent verification	Automated verification using specialized tools and scripts
Detection of Complex Logic Flaws	Effective through human insight	Poor capability	Hybrid approach combining automated and custom scripted tests
Scalability and Repeatability	Limited by human resources	High scalability	Highly scalable due to containerized microservices
Required Expertise	High expertise required	Low to moderate expertise	Moderate; system facilitates semi-automation with expert oversight

Figure 6 illustrates the distribution of AI-generated security recommendations within the developed penetration testing automation platform. The analysis categorizes these recommendations into three distinct outcomes: accepted, modified, and rejected. The quantitative breakdown reveals that 78% of the AI-generated suggestions were accepted without modification, indicating a high level of precision and contextual relevance in the automated assessment process. This outcome suggests that the AI module is effectively aligned with the PTES-based workflow and capable of producing remediation steps that require minimal human intervention.

A further 15% of the AI-generated suggestions were modified prior to implementation. This proportion reflects scenarios in which the automated recommendations were directionally correct but required contextual adjustments to align with specific system architectures, organizational policies, or operational constraints. The need for modification is an expected characteristic in semi-automated security workflows, where the interplay between machine-generated outputs and expert human validation enhances the accuracy and applicability of the final remediation steps.

The remaining 7% of AI-generated suggestions were rejected outright. This segment is indicative of either false positives in the detection pipeline or remediation strategies that were deemed non-viable due to technical, operational, or compliance-related limitations. While relatively small, this proportion highlights the importance of maintaining a human-in-the-loop architecture to ensure that automated processes do not introduce misaligned or potentially disruptive security changes.

From a mathematical perspective, if N represents the total number of AI-generated suggestions, the distribution can be expressed as: Accepted = $0.78 \times N$, Modified = $0.15 \times N$, and Rejected = $0.07 \times N$. For instance, with $N = 500$ suggestions, this corresponds to 390 accepted, 75 modified, and 35 rejected cases. These values provide both a performance indicator of the AI suggestion engine and a benchmark for iterative improvement.

Overall, the observed distribution underscores the efficacy of integrating AI-driven risk detection and remediation guidance into penetration testing processes. The high acceptance rate reflects strong alignment between automated outputs and expert expectations, while the presence of modified and rejected cases justifies the hybrid human-AI validation layer essential in mission-critical cybersecurity operations.

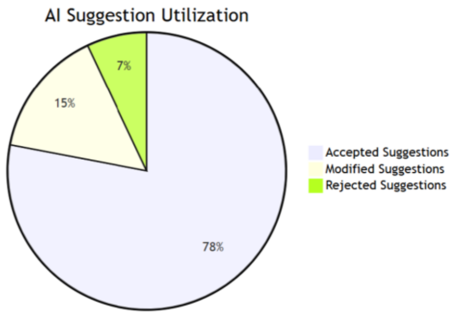

Fig. 6. AI-Generated Security Recommendations: Acceptance Breakdown

The adoption of automation within the PTaaS platform significantly reduces the time required for each penetration testing phase. As shown in Table 2, reconnaissance drops from approximately 10 hours manually to 1.2 hours in a fully automated workflow. Vulnerability scanning experiences a similar reduction, from 15 hours to 1.5 hours, while exploitation time decreases from 8 hours to 1 hour. Even the reporting phase, traditionally time-intensive, is streamlined from 7 hours to 10 min. Overall, the platform achieves substantial efficiency gains, allowing security teams to focus more on analysis and decision-making rather than repetitive manual tasks.

Table 2. Efficiency Metrics

Phase	Manual	Semi-Auto	Full Auto
Reconnaissance	10 hrs	4 hrs	1.2 hrs
Vulnerability Scanning	15 hrs	6 hrs	1.5 hrs
Exploitation	8 hrs	5 hrs	1 hrs
Reporting	7 hrs	5 hrs	10 min

VI. CONCLUSION

This work presents a PTES-aligned PTaaS platform that automates part of the pentesting lifecycle from reconnaissance to reporting. By integrating AI-driven vulnerability analysis, dynamic playbooks for task sequencing, and a central orchestration engine, the platform streamlines and standardizes key test phases. These capabilities significantly accelerate testing workflows and improve consistency, though some steps (such as complex exploitation and validation) still require human expertise. Full end-to-end automation is acknowledged to be a work in progress, and future efforts will focus on expanding automation coverage to the remaining phases. Planned enhancements include enriching the playbooks, incorporating advanced adversarial emulation for more realistic threat scenarios, and further leveraging AI to handle increasingly complex tasks.

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Impact of influencer marketing on the economic performance of a territory: Case of the Dakhla Oued Eddahab Region

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Abstract. This study examines the role of influencer marketing as a strategic communication tool to improve the economic performance of territories, with a particular focus on the Dakhla-Oued Eddahab region in Morocco. While influencer marketing has been widely applied in commercial contexts, its integration into territorial governance remains underexplored. Drawing on theoretical frameworks such as territorial governance, territorial capital, social network theory, and agenda-setting, this research adopts a qualitative methodology based on 12 semi-structured interviews with institutional actors, local entrepreneurs, tourism professionals, and digital influencers. The analysis reveals that influencer marketing contributes to territorial promotion by improving visibility, strengthening cultural identity and stimulating local economic activities, especially in the areas of tourism and entrepreneurship. However, the findings also highlight critical challenges, including distortions in cultural representation, ethical concerns, and unequal access to visibility. The study highlights the need for structured and ethically guided strategies to integrate influencer marketing into territorial development policies, to ensure sustainable, inclusive and culturally sensitive outcomes.

Keywords: influencer marketing, territorial brand, economic performance, governance, Dakhla-Oued Eddahab

I. Introduction

In an increasingly interconnected world, where the visibility and reputation of territories are becoming fundamental levers of competitiveness, territorial marketing strategies are being profoundly renewed by the irruption of digital technology and the rise of influencer practices. Influencer marketing, initially developed in commercial spheres, has gradually established itself as a strategic instrument in public policies, particularly at the local level. By mobilizing individuals or communities with a strong power of prescription on digital platforms, this approach aims to build an attractive territorial image, capture the attention of target audiences and stimulate their engagement. While its effects on symbolic attractiveness are now better documented, its real influence on the economic performance of territories is still little explored in the scientific literature.

This question is particularly relevant in the case of the Dakhla-Oued Eddahab region, a Moroccan territory in full transformation, at the crossroads of national and international strategic issues. With a strong potential for the blue economy, ecotourism and cross-border trade, the region is receiving increasing attention from public and private actors. In this context, influencer marketing campaigns have multiplied in recent years, carried out by local authorities as well as by national and local influencers, with the aim of promoting economic opportunities, investment projects, and local resources. However, beyond the effects of visibility, the central question of their measurable economic efficiency arises: do these schemes really contribute to increasing investment flows, supporting local entrepreneurial dynamics, or improving territorial economic performance indicators?

This research proposes to question the impact of influencer marketing on the economic performance of a territory, by adopting both a conceptual and empirical approach focused on the Dakhla-Oued Eddahab region. It is part of a twofold perspective: on the one hand, to understand the mechanisms by which influence strategies participate in the construction of a territorial competitive advantage; on the other hand, to assess the perceived and measurable effects of these strategies on the economic dimensions of local development, such as private investment, tourist attractiveness, employment or value creation.

In order to respond to these issues, the study is based on a critical review of theoretical models relating to territorial governance, public marketing and local economic performance, complemented by a field survey conducted among institutional, economic and communication actors in the region. The objective is to highlight the possible links between communication of influence, territorial strategy and economic development, while identifying the success factors, limits and conditions of sustainability of these systems.

Through this approach, this contribution intends to enrich the scientific reflection on the new instruments of territorial management in the digital age, and to provide operational avenues to local decision-makers for the efficient use of influencer marketing as a vector of inclusive and sustainable economic development.

1. Influencer marketing in the territories

In the digital age and participatory communication, influencer marketing has established itself as a central lever in communication strategies, not only for companies, but also for public actors. This concept refers to all actions that consist of mobilizing people with a strong audience and recognized credibility with a target audience, the "influencers", in order to transmit messages, enhance an image or encourage behavior. While its first applications were mainly commercial, influencer marketing is now expanding rapidly in non-market sectors, particularly in local and territorial public policies.

In this context, many local authorities, development agencies, or tourist offices are taking up this practice to increase their visibility, promote their territory and strengthen their attractiveness. These influencer campaigns are often focused on promoting the cultural, economic or natural assets of a territory, through visual content, stories of experience or partnerships with local or national influencers. The objective is no longer just to communicate, but to capture attention, generate support and engage citizens and visitors alike in a dynamic of collective enhancement of the territory.

1.1. Definition of territorial marketing

Influencer marketing is a communication strategy based on the use of social influence exercised by individuals or entities with a loyal, engaged and targeted audience, particularly on digital platforms. These individuals, called influencers, are perceived by their community as trusted prescribers, capable of recommending, guiding or shaping the opinions and behaviors of their followers (Freberg et al., 2011). Unlike traditional forms of institutional communication, influencer marketing is part of a logic of relational intermediation, where proximity, authenticity and interaction are central to the process of persuasion.

In the territorial field, this strategy is manifested by the mobilization of influencers, whether local, national or thematic, to strengthen the visibility of a territory, enhance its image, and attract specific audiences (tourists, investors, citizens, etc.). Influencer marketing then becomes a territorial communication tool, used by local authorities, development agencies, or tourist offices, in order to disseminate attractive narratives, create positive representations of the territory and engage communities in a logic of identity co-construction (Bastos & Casais, 2019; Kavaratzis & Hatch, 2013).

This approach presupposes a paradigm shift in the relationship between public institutions and populations. It breaks with top-down and unilateral communication, promoting horizontal, participatory and immersive communication. It can thus contribute to building a territorial brand image perceived as more credible and engaging, by relying on influential figures who embody the values, specificities or ambitions of the territory. In this respect, influencer marketing is fully in line with the contemporary logics of territorial marketing and government through image, where reputation and differentiation become strategic resources for territories (Hudson & Thal, 2013; Lucarelli & Berg, 2011).

However, its use in the public sector also raises ethical, methodological and operational challenges: selection of influencers, transparency of partnerships, measurement of impact, adequacy with public values, or reputational risk management. These questions invite us to think of influencer marketing not as a simple promotional tool, but as a strategic vector integrated into contemporary territorial governance.

1.2. The use of influencer marketing by local actors

At a time when territories are engaged in increased competition to attract investment, tourists, talent and projects, territorial actors are adopting innovative communication strategies, of which influencer marketing is becoming a privileged lever. Initially used in the commercial sectors, this system is now integrated into local public policies, in particular by local authorities, development agencies, tourist offices, as well as certain parapublic structures (Bastos & Casais, 2019; Kavaratzis & Ashworth, 2008).

These institutions rely on local or national influencers to promote the image of their territory, enhance its cultural, natural or economic specificities, and engage citizens in a dynamic of participation and identity appropriation. For example, many regions and cities collaborate with content creators on Instagram, YouTube or TikTok to spread positive representations of the territory, through immersive narratives and authentic experiences (Lucarelli & Berg, 2011). These partnerships make it possible to reach younger and connected audiences, who are often not very receptive to traditional institutional campaigns (Hudson & Thal, 2013).

In a logic of territorial marketing, these actions aim to strengthen the overall attractiveness of the territory: attract new visitors, support local economic dynamics (tourism, crafts, agriculture, etc.), or encourage the installation of new residents or investors. Digital influence thus becomes a tool for storytelling the territory (territorial storytelling), based on emotion, proximity and recommendation perceived as authentic (Kalandides, 2011).

However, this use raises important issues: choice of influencers, alignment of the discourse with public values, measurement of the real economic impact, and ethical framework for collaborations. The increasing professionalization of systems is thus encouraging public actors to adopt more structured strategies, integrating influencer marketing into a broader vision of communication governance and territorial enhancement (Zenker & Erfgen, 2014).

1.3. Objectives of influencer marketing

In a territorial context marked by rapid economic, social and environmental changes, local authorities are faced with the need to rethink their communication strategies to better meet the varied expectations of target audiences (Durand, 2021). These strategies must not only reflect the territory's own identity, but also promote the creation of a lasting link with residents, visitors and institutional partners, in a logic of shared territorial development (Martin & Leroy, 2018). Territorial communication thus plays a key role in strengthening social cohesion and improving the visibility of the territory, particularly in the face of increased competition between different regions or cities to attract investment and tourism (Bazin, 2020).

In addition, the diversification of communication media, particularly with the emergence of social networks and digital platforms, as well as the growing information saturation, make it essential to adopt innovative and multichannel approaches (Lemoine & Petit, 2019). In this context, captivating the attention of different audiences is a major challenge, as is promoting active involvement and real participation that goes beyond the simple passive reception of the message (Moreau, 2020). These challenges require the implementation of integrated and coherent strategies, capable of establishing genuine dialogue and interaction, which are essential elements for the revitalization, sustainability and enhancement of the territory in the long term.

- Enhancing the image of the territory: Enhancing the territorial image is a strategic approach aimed at building and disseminating a positive, distinctive and coherent representation of the territory to its various stakeholders. This enhancement is based on the identification and highlighting of the intrinsic assets of the territory, whether cultural, historical, economic, environmental or social (Durand, 2021). It is a real identity construction that contributes to forging a solid reputation, an essential factor in the competition between territories to attract tourists, investors and new residents. In addition, a valued image reinforces the sense of belonging of the inhabitants, thus consolidating social cohesion. Valorisation requires controlled and multi-channel communication, integrating traditional media, social networks, but also symbolic events and territorial storytelling actions. This integrated approach makes it possible to generate a positive and lasting perception, increase awareness and promote the overall attractiveness of the territory (Lemoine & Petit, 2019). In addition, it prepares the territory to meet the expectations of diverse audiences and to anticipate future socio-economic developments.

- Attract the attention of target audiences: In a saturated and competitive information environment, attracting attention is a major challenge for any territorial communication strategy. This step is essential to capture the initial interest of target audiences, citizens, visitors, companies, institutional partners, and thus guarantee the visibility of the messages and initiatives carried by the territory (Martin & Leroy, 2018). The effectiveness of this attraction is based on the use of innovative, interactive tools and content adapted to the specificities of the public and communication media. This includes the creation of creative campaigns, the use of digital marketing, dissemination via social networks, as well as the organization of attractive and mediatized events. In addition, the ability to arouse curiosity and provoke a positive emotion plays a decisive role in memorizing messages and building a dynamic territorial image. Attracting attention is not limited to the simple exhibition, but also aims to establish a lasting relationship with the public, a sine qua non condition for their subsequent engagement.

- Engaging citizens and visitors: The engagement of citizens and visitors is a key lever in territorial governance and the sustainable development of the territory. The aim is to actively involve these actors in local life, by promoting their participation in decisions, projects and initiatives that concern them directly (Moreau, 2020). This participatory approach helps to strengthen the sense of belonging, to increase the legitimacy of public policies and to improve the quality of decisions by better taking into account the needs and expectations of stakeholders. To this end, engagement mechanisms can take various forms: public consultations, collaborative workshops, participatory digital platforms, citizen events, etc. Beyond the democratic dimension, the commitment creates a collective and solidarity dynamic that stimulates social cohesion and territorial resilience. In addition, committed citizens and visitors become ambassadors of the territory, positively spreading its image to the outside world and contributing to its influence. This sustainable mobilization is therefore an essential driver for attractiveness, competitiveness and territorial vitality.

2. Economic performance of a territory

The economic performance of territories is a central issue in a context marked by globalisation, technological transformations and competition between regions and cities (Dupont & Lefèvre, 2019). This notion refers to the ability of a territory to effectively mobilize its resources to generate wealth, create jobs and attract investment. It goes beyond the simple measurement of local gross domestic product (GDP) to integrate qualitative dimensions such as economic diversification, innovation and resilience to economic change (Martin, 2020). Consequently, territorial economic performance is a key indicator to guide public policies and promote balanced and sustainable development.

In addition, the increasing complexity of territorial dynamics requires a multidimensional and integrated approach to economic performance (Garnier, 2018). The latter takes into account not only the economic results, but also the structural, social and environmental factors that influence the competitiveness and attractiveness of territories. In addition, the evaluation of economic performance is based on the mobilization of local actors and the quality of the infrastructure, services and living environment offered (Lemoine & Petit, 2019). Thus, understanding and measuring territorial economic performance is a major challenge to ensure the coherence of territorial strategies and support local development in a context of increased competition.

2.1. Definition of the economic performance of a territory

Territorial economic performance refers to the ability of a territory to generate, in a sustainable and balanced way, dynamic economic activity, to create jobs and attract investment, while ensuring the quality of life of its inhabitants and the sustainability of its resources (Dupont & Lefèvre, 2019). This notion, although often associated with traditional quantitative indicators such as regional gross domestic product (GDP), the employment rate or the level of investment, also encompasses essential qualitative dimensions. These include the diversification of economic sectors, innovation, the competitiveness of local businesses, as well as the territory's ability to resist and adapt to economic, social and environmental changes (Martin, 2020). Indeed, an economically efficient territory is not limited to producing wealth, but must also guarantee inclusive growth that respects social and environmental balances.

In addition, territorial economic performance is closely linked to local governance and public policies implemented by territorial actors, whether public, private or associative (Garnier, 2018). The effectiveness of these strategies conditions the territory's ability to strengthen its attractiveness, in particular through the development of adapted infrastructure, the training of a qualified workforce, and the establishment of a framework conducive to innovation and entrepreneurship (Lemoine & Petit, 2019). It also implies an integrated approach, taking into account the interactions between the economic, social and environmental dimensions to promote sustainable development. Thus, territorial economic performance is part of a systemic perspective, which considers not only immediate results, but also the resilience of the territory in the face of crises, social cohesion and the ability to adapt to global transformations (Bazin, 2020). This multidimensional approach makes it possible to guide local strategic planning and to better respond to the complex issues facing contemporary territories.

2.2. Key indicators of economic performance

The evaluation of territorial economic performance is based on several key indicators that make it possible to measure the vitality and dynamism of a territory. Among these indicators, investment occupies a central place, as it reflects the region's ability to attract capital, whether public or private, necessary for the development of infrastructure, equipment and economic activities (Dupont & Lefèvre, 2019). Investment is therefore an essential lever for stimulating economic growth and strengthening local competitiveness.

Employment is another fundamental indicator, directly linked to territorial economic performance. The employment rate, the unemployment rate and the quality of the jobs offered are key elements that reflect the territory's ability to generate professional opportunities for its inhabitants (Martin, 2020). In addition, tourism represents a strategic sector for many territories, particularly those with cultural, natural or heritage assets. The flow of tourists, the number of visitors to the sites and the revenue generated are valuable indicators for assessing the attractiveness and economic performance of a region (Garnier, 2018).

In addition, the creation of companies is a dynamic indicator that testifies to the entrepreneurial spirit and the capacity for innovation of the territory. A high rate of business creation reflects an environment conducive to economic initiative, the diversity of activities and the generation of new jobs (Lemoine & Petit, 2019). These combined indicators offer a global and multifaceted vision of territorial economic performance, making it possible to guide public policies and development strategies adapted to local specificities.

Table 1: Overview of the major indicators of local economic performance

Indicator	**Description**	**Importance**	**Typical data sources**
Investment	Amount of public and private capital injected into the territory for economic development	Stimulates growth, modernizes infrastructure and supports innovation (Dupont & Lefèvre, 2019)	Local statistics, investment agencies
Employment	Employment rate, unemployment rate, quality of jobs offered	Key indicator of economic health and social cohesion (Martin, 2020)	National statistical services, local surveys
Tourism	Number of visitors, overnight stays, tourism revenue	Measures the attractiveness of the territory and its economic benefits (Garnier, 2018)	Tourist Offices, Ministries of Tourism
Setting up a business	Number of new businesses created in the territory	Bears witness to entrepreneurial dynamics and local innovation (Lemoine & Petit, 2019)	Commercial registers, chambers of commerce

Source: prepared by the authors

The combined analysis of these indicators makes it possible to obtain an overall and nuanced assessment of the economic performance of a territory. While investment and business creation reflect the region's ability to innovate and develop, employment directly reflects the social benefits of this growth. Tourism, on the other hand, is often a strategic sector for regions with specific natural or cultural assets, contributing to economic diversification and the generation of additional income. Together, these indicators provide a valuable analytical framework to guide political and economic decisions, tailoring strategies to the specific strengths and weaknesses of each territory.

2.3. Towards integrated territorial performance: when the key factors align

The performance of a territory, whether economic, touristic or social, depends largely on several interconnected factors. Among these, infrastructure plays a key role. Indeed, modern and adapted infrastructure facilitates the mobility of people and goods, improves access to services, and is an essential lever for economic development (Dupont, 2020). For example, the quality of transport networks, telecommunications and public facilities directly influences a territory's ability to attract investors and visitors (Martin & Leclerc, 2019).

Moreover, the image of the territory is an immaterial but just as decisive factor. The image perceived by the target audiences, whether they are tourists, entrepreneurs or residents, conditions their investment or stay decisions (Durand, 2018). A positive image, built through targeted communication campaigns and territorial branding actions, helps to strengthen the notoriety and reputation of the territory (Smith & Brown, 2021).

Territorial attractiveness, often analysed as a combined result of quality of life, economic opportunities, and the services offered, is another key factor. According to Lefebvre's (2017) study, the most attractive territories know how to combine a business-friendly environment with a cultural and social richness appreciated by the population. This attractiveness has a direct influence on migratory flows, investments, and therefore the overall performance of the territory.

In addition, institutional and promotional communication plays a strategic role in the development of the territory. Effective communication, integrating digital tools and social networks, makes it possible to quickly disseminate a coherent and rewarding image, to engage local and external stakeholders, and to strengthen the visibility of the territory in a competitive environment (Nguyen & Thomas, 2022).

Thus, territorial performance is the result of a synergy between these factors: solid infrastructure, a well-groomed image, assertive attractiveness and dynamic communication, which must be integrated into a coherent overall strategy to maximize results.

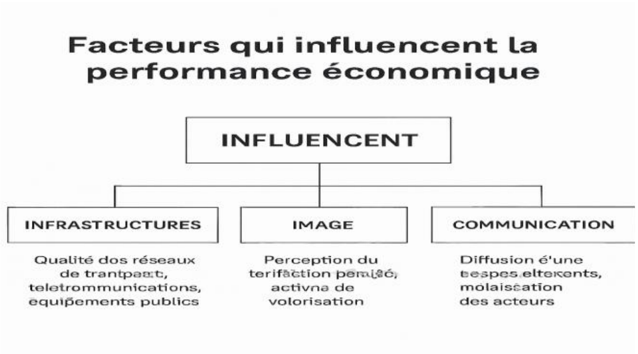

Fig 1: Integrated diagram of territorial performance factors

Source: prepared by the authors

Moreover, it is important to emphasize that these factors do not operate in isolation, but are interdependent and influence each other. For example, high-performance infrastructure can enhance the image of a territory by facilitating access and improving user comfort, which in turn increases its attractiveness to investors and tourists. At the same time, well-orchestrated communication not only promotes the territory's infrastructure and assets, but also manages its reputation in times of crisis or transformation, thus strengthening the trust of stakeholders (Healey, 2006). This interdependence underlines the need for an integrated and coordinated approach to ensure sustainable and resilient territorial performance.

Moreover, in a context of globalization and increased competition between territories, the ability to innovate in the management of these factors is becoming a major competitive advantage. The integration of new technologies into infrastructure, the use of innovative digital communication strategies, and the development of participatory initiatives that promote public support are all elements that make it possible to adapt territorial performance to contemporary issues (Florida, 2002; Nguyen & Thomas, 2022). Thus, the success of a territory also depends on its ability to evolve and anticipate the expectations of the various stakeholders, by implementing flexible and proactive policies.

3. Literature review on influencer marketing and economic performance

The study of the interactions between influencer marketing and territorial economic performance mobilizes several theoretical frameworks from the social sciences, communication and territorial management. These approaches make it possible to understand the mechanisms by which influence strategies contribute to the enhancement and dynamism of territories.

3.1. The theory of territorial governance

Territorial governance is defined as a mode of coordination and regulation involving the collaboration of multiple public, private and associative actors working at different scales within a given territory (Pierre & Peters, 2000). This theory goes beyond classical administrative management by integrating participatory, negotiated and often collaborative processes (Healey, 1997). It highlights the ability of a territory to mobilise and organise its human, institutional and material resources to face economic and social challenges.

In this context, influencer marketing appears to be a strategic lever for territorial governance. By facilitating targeted communication and the mobilisation of stakeholders, it makes it possible to strengthen the legitimacy of local initiatives and to amplify the visibility of the territory. By bringing together different actors (local authorities, companies, influencers), this strategy promotes the emergence of a common and shared vision, an essential condition for sustainable and coherent economic development. For example, influencer campaigns make it possible to promote local projects, attract new investors or tourists, and create a sense of belonging among the population. Thus, influencer marketing is not limited to a simple communication operation, but is part of an integrated governance process, where participation and cooperation are key factors in economic performance (Pierre, 2011).

3.2. The theory of territorial capital

The theory of territorial capital postulates that the attractiveness and competitiveness of a territory are based on a set of specific resources, tangible and intangible, which form a single territorial capital (Deschamps, 2003). This capital includes infrastructure, natural resources, but also intangible dimensions such as the quality of human capital, the institutional network and reputation. The ability of a territory to develop this capital determines its success in economic competition.

Influencer marketing intervenes here as a vector for enhancing the intangible capital of the territory. By disseminating a positive and differentiated image, it contributes to building an attractive territorial identity, thus promoting the recognition and trust of external actors. Through the creation of engaging content and the mobilization of local or national influencers, this strategy makes it possible to tell a strong territorial story, which highlights cultural, economic and social assets. This narrative helps to differentiate the territory in a competitive environment, attracting investment, talent and visitors (Becattini, 1991). Therefore, influencer marketing is a strategic tool that activates intangible territorial capital to generate concrete effects on local economic performance.

3.3. Social network theory

Social network theory highlights the importance of interpersonal and interorganizational relationships in the flow of information, innovation, and reputational building (Granovetter, 1973). It emphasizes that individuals or groups occupying central positions in a network of influence have significant power to guide opinions and behaviors.

In the territorial context, digital influencers represent those strategic nodes capable of amplifying messages on social platforms, creating engagement and facilitating the rapid flow of information. Influencer marketing leverages this structure by identifying and mobilizing these actors to maximize the reach and impact of territorial campaigns. By stimulating community participation and buy-in, it helps to strengthen social cohesion and local economic dynamics (Burt, 2005). In addition, the dissemination of a coherent and positive image via these networks contributes to the construction of a solid territorial reputation, a key factor in attractiveness and therefore in economic performance.

3.4. The theory of territorial agenda-setting

Stemming from the pioneering work of McCombs and Shaw (1972), agenda-setting theory analyzes the role of the media and influential actors in the selection and prioritization of subjects of collective attention. Applied to the territory, it explains how certain themes, images or territorial projects can be put forward in order to shape public perception.

Influencer marketing is a powerful instrument in this process. Through targeted communication strategies, it makes it possible to create and disseminate a coherent territorial agenda, highlighting key themes such as sustainable development, innovation, or tourist attractiveness. This framing influences not only the way in which the public perceives the territory, but also their behaviour, particularly in terms of investment or visits (Parsons, 2013). By controlling the media and digital agenda, influencer marketing helps to orient the image of the territory favourably, which has a direct impact on its economic performance.

4. Research Methodology

Faced with the increasing complexity of digital devices in territorial governance, this research adopts a qualitative approach in order to understand the social, discursive and strategic dynamics that underlie the use of influencer marketing in a specific territorial context. This methodological choice is motivated by the very nature of the problem, which concerns processes that are difficult to quantify: subjective perceptions, social representations, legitimation logics, identity narratives and visibility dynamics. Rather than seeking to measure the impact of influence campaigns on economic indicators in a linear manner, the study aims to understand how territorial actors perceive, mobilise and evaluate these strategies, in a logic of social construction of development. The Dakhla-Oued Eddahab region, chosen as a field of study, represents a relevant case of a changing territory, strongly exposed to the logic of attractiveness, image and digital storytelling. The objective is therefore to question the symbolic and institutional effects of influence campaigns on the territorial fabric, beyond the simple measurable economic benefits.

The production of empirical data was based on semi-structured interviews conducted with a reasoned sample of strategic actors involved in governance, communication or the territorial economy. This sample includes: local elected officials responsible for territorial marketing or tourism, digital influencers who have collaborated on campaigns in the region, managers of public communication agencies, entrepreneurs benefiting from territorial promotion initiatives, as well as representatives of parastatal development organizations. These interviews, conducted between March and May 2025, collected rich data on the representations, motivations and strategies of the actors concerned. Each interview was prepared using a thematic interview guide covering several axes: the perception of influencer marketing, the forms of collaboration between institutions and influencers, the perceived effects on territorial attractiveness, the impact on civic engagement and the ethical issues raised. All interviews were recorded, transcribed in full and anonymized in order to respect the principles of research ethics. In addition, a content analysis was conducted on a selection of digital publications (Instagram posts, YouTube videos, institutional reports) from campaigns targeted in Dakhla, with the aim of studying the visual narratives, emotional registers and elements of territorial storytelling mobilized.

The data analysis followed an inductive thematic analysis method, drawing on the principles of Grounded Theory, in order to surface the categories from the data itself rather than from a predefined framework. After a floating reading and an initial free coding of the interviews, recurring categories were identified, such as: perceived image of the territory, authenticity of influencers, instrumentalization of emotion, legitimacy of public policies, identity issues, or ethical contradictions. These themes were then organized into transversal analytical axes allowing to cross points of view according to the status of the actors (institutional, private, communicative). This analysis has made it possible to identify narrative and strategic logics that guide the way in which the territory is told, sold and governed through the tools of influence. By restoring the words of the actors and highlighting the diversity of uses, resistances and appropriations of influencer marketing, this methodology aims to enrich reflections on contemporary changes in territorial public communication. It also offers a critical look at the tensions between visibility, economic performance and local anchoring in emerging territories.

5. Empirical results

As part of this research, 12 semi-structured interviews were conducted with actors directly involved in the territorial dynamics of the Dakhla-Oued Eddahab region. The sample was constructed according to a logic of diversification of profiles, in order to cross institutional, economic, communicative and community points of view. It includes local elected officials, influencers active on digital platforms, managers of territorial communication agencies, entrepreneurs, as well as players in the tourism sector. These interviews, which lasted between 35 and 60 minutes, were conducted on the basis of an interview guide structured around four main analytical axes. The first axis aimed to explore the perception of influencer marketing as a strategic tool in territorial development. The second focused on the practices and modalities of local use of this approach: types of content disseminated, privileged digital channels, profiles of influencers mobilized. The third axis focused on the perceived effects of influencer marketing on the region's economic performance, particularly in terms of attractiveness, investment, tourism or entrepreneurship. Finally, a fourth axis critically addressed the issues, limits and conditions of legitimacy of these practices, in particular through questions of ethics, authenticity and local anchoring. This structuring made it possible to collect rich and nuanced qualitative data, while ensuring consistency of analysis between the different interviews conducted.

As part of this qualitative study conducted in the Dakhla-Oued Eddahab region, an interview guide was structured around four thematic axes, the first of which focuses on the perception of influencer marketing as a territorial communication tool. In order to summarize the recurring elements mentioned by the twelve participants, a word cloud was generated from all the responses relating to this axis. This technique makes it possible to identify the most frequently used terms, thus offering an immediate reading of salient concepts, while facilitating the identification of central discursive dimensions.

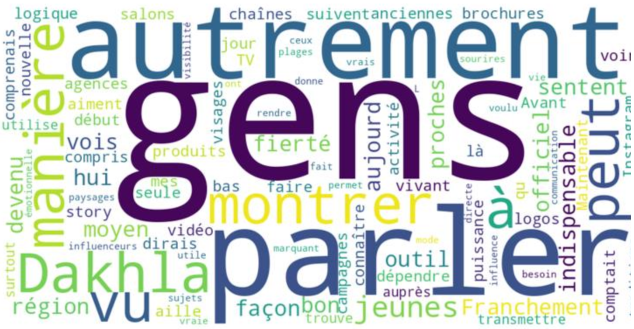

Fig 2: Word cloud from responses related to the perception of influencer marketing (Axis 1)

Source: JupyterLab

Visual analysis of the word cloud reveals a high density around the terms "Dakhla", "video", "region", "influence", "traditions", "communication", "youth", "tool" and "visibility". These occurrences suggest a convergence of discourses around an identity and promotional enhancement of the territory through embodied digital media. The word "Dakhla", central and very present, indicates a strong territorial anchoring in the respondents' representations, reflecting an appropriation of influencer marketing as a vector of local development. The term "video" refers to the formats favoured in the influence practices observed, particularly via platforms such as Instagram, TikTok or YouTube, which are frequently mentioned during interviews.

The terms "traditions" and "communication" show a tension between cultural authenticity and the need for contemporary mediation. This shows a desire to transmit the local cultural heritage through a visual language accessible to the external public, while avoiding excessive folklorization. Finally, the presence of the word "youth" suggests that influencer marketing is perceived not only as an external lever for tourist attractiveness, but also as a tool for generational engagement, mobilizing young people as relays or digital ambassadors of the territory.

The second axis of the interview guide focuses on the uses and concrete strategies put in place by local actors to integrate influencer marketing into their territorial enhancement actions. Participants were invited to describe the mechanisms they mobilize, the collaborations they have engaged, as well as the tools and platforms they use. In order to summarise the emerging content of these stories, a word cloud was generated from the responses collected. This visualization makes it possible to identify the dominant practices, the platforms mobilized and the target audiences mentioned in the corpus.

Fig 3: Word cloud of responses relating to influencer marketing uses and strategies (Axis 2)

Source: JupyterLab

The word cloud relating to this axis highlights a lexical concentration around the terms "campaign", "videos", "creators", "TikTok", "YouTube", "young people", "content", "influencer" and "market". This recurrence reflects a structuring of practices around short and immersive digital formats, and a desire for strategic diversification according to the target audiences. The word "videos" occupies a central position, confirming the importance given to the audiovisual medium in the transmission of territorial narratives. The TikTok and YouTube platforms are frequently mentioned, which reflects a double strategic positioning: on the one hand, TikTok's viral immediacy to increase visibility; on the other, YouTube's ability to offer more in-depth content to promote narrative anchoring.

The high occurrence of the term "young people" suggests two complementary dynamics: young people are considered both as privileged targets of campaigns and as actors producing content. The commitment of the latter as local content creators is also suggested by the words "trained", "daily" or "testimonies", which reflect an inclusive approach in influence strategies. Finally, the presence of words such as "campaign", "projects", "market", "creators", "collaboration" signals a progressive professionalisation of uses, where the logic of co-creation of value is at the heart of the systems. The adaptation of content according to geographical or cultural markets (Moroccan, Spanish, ecological, Amazigh, Saharawi) reinforces this reading of a local strategy articulated with global logics of influence.

The third axis of the qualitative survey aims to explore the perceived economic effects of influencer marketing actions on local activities in the Dakhla-Oued Eddahab region. Through concrete stories, the participants described the impact observed on their bookings, sales, notoriety or professional relationships. In order to synthesize the lexical recurrences resulting from these testimonies, a word cloud was generated from all the responses, highlighting the terms most frequently used to evoke economic benefits.

Fig 4: Word cloud of responses relating to the economic effects of influencer marketing (Axis 3)

Source: JupyterLab

The analysis of the word cloud reveals a strong presence of lexemes related to commercial performance, such as "reservations", "customers", "sales", "visibility", "profile", "workshop", "contract", "notoriety", "orders" or "partnerships". These terms indicate that local players perceive influencer marketing as a direct or even measurable lever for economic activation. The recurrence of the word "bookings" reflects a clear impact for tourism and accommodation professionals, often mentioned as the first beneficiaries of digital campaigns. This data is reinforced by the terms "customers" and "orders", which refer to actual commercial interactions, sometimes even internationally, as illustrated by the mention of foreign buyers. The high occurrence of "visibility" and "notoriety" signals a recognition of less immediately monetary, but just as strategic, effects in terms of brand image. These symbolic dimensions appear to be closely linked to the digital tools used, in particular social profiles and videos shared by influencers, contributing to increased media coverage of the territory and its economic actors. Finally, the words "partnerships" and "contract" suggest a structural dimension of impact, with the possibility of more sustainable institutional or commercial opportunities. This suggests that influencer marketing, beyond ephemeral visibility, is part of medium-term local development logics, in particular through the networking of actors and the gradual professionalization of initiatives.The fourth axis of the interview guide aimed to question the reservations, criticisms and concerns that can be raised by the uses of influencer marketing in a territorial context. Unlike the previous axes focused on uses or positive impacts, this part aimed to explore the grey areas, cultural tensions, and ethical issues felt by the local actors interviewed. The word cloud below summarises the dominant lexical fields mentioned during the interviews concerning the risks of drift or instrumentalisation of the territory.

Fig 5: Word cloud of responses related to the limits and legitimacy of influencer marketing (Axis 4)

Source: JupyterLab

The analysis of the word cloud highlights terms that are suggestive of structural or symbolic criticisms, including: "culture", "image", "abuse", "problem", "reality", "message", "tourists", "distorted", "authorities", "expectations", or "inequality". These occurrences bear witness to a reflexive discourse of local actors, aware of the ambivalent effects of influence strategies. The word "culture" appears to be central: it refers to a fear of distortion or oversimplification of local identity, often staged in a spectacular or superficial way to seduce an external audience. This concern is reinforced by the emergence of words such as "distorted" or "reality", suggesting that the image projected by certain content is sometimes out of step with authentic local life.

The notions of "abuse" and "problem" reflect a lack of regulation or ethical framework, evoked through stories where some influencers have acted opportunistically, without respect for cultural or economic contexts. The word "authorities" reflects this lack of institutional supervision or participation, making initiatives sometimes disorganized or even counterproductive. Finally, the terms "inequality", "expectations", "audience" and "message" refer to more systemic dimensions: campaigns can benefit some actors to the detriment of others, create unrealistic expectations among visitors, or convey standardised discourses that are not very representative. Influencer marketing is thus perceived as an ambivalent opportunity, both useful and potentially threatening to the balance of the territory.

As part of this qualitative study conducted in the Dakhla-Oued Eddahab region, four thematic axes structured the interview guide and guided the analysis of the data collected. Each axis corresponds to a specific dimension of influencer marketing as a tool for territorial enhancement: its perception by local actors, its concrete uses and associated strategies, its perceived economic effects, as well as the limits and ethical questions it raises. The table below provides a comparative summary of the results for each of the axes. It relates the dominant themes identified in the speeches, the recurring keywords from the word clouds, as well as the main tensions or crossovers that have arisen between the different dimensions. This perspective allows us to better understand the articulation between the representations, practices and issues associated with influencer marketing in a specific territorial context.

Table 2: Summary table

Axle	Main themes	Dominant keywords	Tensions or crossings
Axis 1: Perception	Emotional communication, proximity, cultural enhancement	Dakhla, video, traditions, young people, faces	Tensions with folklorized representations (Axis 4)
Axis 2: Uses & strategies	Strategic Targeting, Local Creators, Short/Long Formats	campaign, creators, TikTok, YouTube, youth, markets	Exclusion of certain groups, risk of appropriation (Axis 4)
Axis 3: Economic impacts	Increase in bookings, visibility, concrete benefits	Reservations, customers, visibility, notoriety, partnerships	Need for more rigorous evaluation (Axis 4)
Axis 4: Limits and legitimacy	Risk of deformation, inequalities of access, lack of regulation	culture, abuse, image, tourists, message, distorted, authorities	Need for an ethical and institutional framework (strengthens Axis 2)

The qualitative analysis conducted on the basis of interviews conducted in the Dakhla-Oued Eddahab region has made it possible to identify a nuanced and rich vision of the role that influencer marketing plays in the dynamics of territorial communication and local development. Through the four axes explored, the data reveal a progressive appropriation of this tool by local actors, often with creativity and adaptability, but also with caution and critical thinking. The results show that influencer marketing is perceived positively as a vector of identity enhancement, economic promotion and generational commitment. It now appears to be integrated into structured strategies, with targeted choices of platforms, content, and audiences. However, this appropriation is accompanied by a growing need for regulation, supervision and evaluation, in order to avoid abuses linked to overrepresentation, cultural distortion, or unequal access to visibility. All the responses therefore highlight a structuring ambivalence: while influencer marketing represents an opportunity for positive transformation for territories, it in return implies increased vigilance in terms of ethics, representativeness and sustainability of practices. This double reading opens the way to a broader reflection on the social and cultural legitimacy of new digital communication tools in local development policies.

Conclusion

At the end of this exploratory research on the impact of influencer marketing in the territorial context of Dakhla-Oued Eddahab, several major lessons can be drawn. The study demonstrated that influencer marketing is not simply a passing digital trend, but a strategic communication tool with perceptible effects on the visibility, attractiveness and economic performance of a territory. The local actors we met, craftsmen, entrepreneurs, institutions or creators, testify to a growing adoption of these practices, often in connection with a desire to enhance local identity, diversify audiences and stimulate economic activity. The use of short formats (Reels, Stories, TikTok) or more narrative content (YouTube, testimonials) reveals a creative adaptation to the contemporary codes of digital communication, while maintaining a strong link with the cultural specificities of the territory. However, this dynamic is accompanied by significant limitations and challenges, particularly in terms of regulation, coherence of messages, protection of intangible heritage and equitable access to visibility. The risk of folklorization, commercial over-representation or disconnection with local realities calls for a better institutional structuring of these initiatives, by promoting training, ethical supervision and the evaluation of the benefits. Ultimately, influencer marketing appears to be a potential lever for territorial development, provided that it is thought of not only as a promotional tool, but as a means of local expression, inclusion and co-construction of identity. The case of Dakhla thus illustrates the possibilities of a deep-rooted digital communication, capable of articulating modernity and authenticity in a logic of sustainable and participatory development.

Although this research sheds significant light on the uses and effects of influencer marketing in a specific territorial context, it has a number of methodological and empirical limitations that should be rigorously highlighted.

On the one hand, the study is based on a qualitative approach based on a small sample of twelve semi-structured interviews, which limits the generalization of the results.

While the data collected offer a discursive richness that allows us to identify strong trends, they remain anchored in a particular socio-economic and cultural context, that of the Dakhla-Oued Eddahab region. A greater diversity of profiles, in particular by integrating more public decision-makers, tourists or territorial marketers, could make it possible to broaden the analysis. On the other hand, the research focused on the declarative perceptions of the actors, without cross-referencing with quantitative performance data (traffic, sales, measured awareness, engagement rate, etc.). The lack of methodological triangulation thus constitutes a limit to the robustness of the advanced correlations between influencer marketing and economic performance.

In terms of research perspectives, several avenues deserve to be explored. In particular, it would be relevant to conduct an interregional comparative study (for example with other regions of Morocco or North Africa) in order to identify differentiated models of appropriation of influencer marketing. In addition, a mixed approach integrating quantitative methods (surveys, analysis of social data, economic indicators) could make it possible to measure the real impact of these strategies more precisely. Finally, at a time when digital platforms are rapidly evolving, future research could focus on emerging forms of territorial influence (micro-influencers, local ambassadors, participatory tourism), but also on the ethical, cultural and political issues raised by the intensification of this digital visibility in territories.

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Enhancing Malware Anomaly Detection through Static Analysis, Machine Learning, and Similarity Hashing

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Abstract. This paper presents a hybrid malware detection method that combines machine learning, fuzzy hashing, and static analysis. The method uses similarity-based hashing to find possible variants after extracting structural and semantic characteristics from executable files, such as metadata and cryptographic signatures. Models for classification that can differentiate between dangerous and benign software are trained using these enriched features. Static analysis and clever algorithms together improve resilience against obfuscated malware while maintaining accuracy in real-world detection tasks, according to experimental evaluation.

I. Introduction

Endpoint Detection and Response (EDR) solutions are the linchpin of cyber security, providing ongoing monitoring and analysis of endpoint activity that not even the most sophisticated attackers can evade. They have evolved to become more advanced at data gathering, behavioral analysis and automated response. But the massive amount and intricacy of endpoint data often make separating out unwanted behavior from the unwanted nearly impossible without the aid of smarter analytic methods. Detecting malware on endpoints and doing so efficiently remains a key challenge with rapidly changing threat landscape. The complexity of modern assaults highlights the shortcomings of traditional static signature-based techniques. A multilayer detection approach that combines sophisticated static analysis, machine learning's predictive power, and hash comparison's dependability shows promise as a means of boosting system security. Static analysis allows for the detection of questionable patterns and structures without execution, whereas machine learning offers the capacity to recognize abnormalities and unknown risks. Lastly, hashing guarantees the quick detection of malware that has already been cataloged. The goal of this tactical combination is to greatly strengthen EDRs' security posture against present and potential threats. Machine learning (ML) has been recently proposed as a promising approach to overcome such limitations, by enabling the derivation of complex patterns from large collections of data and the identification of weak signals of malicious activities. Varying supervised and unsupervised learning methods, including decision trees, support vector machines, and deep learning models, have been utilized to find anomalies, classify malware, and forecast threats. While the approaches of this kind greatly elevated the detection precision level, they come with the corresponding liabilities of the type of excessive false positive rates or uninterpretability that make it impossible for the security analysts to comprehend the rationale of the warning. Graph analysis can aid in focusing investigation and detecting complex patterns of harmful behaviors. However, generating and analysing large-scale, dynamic graphs from endpoint data presents major computational and scalability issues. Evolution of EDR Techniques. As cybersecurity threats became more sophisticated, EDR solutions underwent fundamental modifications.

Table 1. Progression from early reactive approaches to current proactive defense paradigms :

Years	Techniques Used in EDR
2005-2010	Signatured based Detection, Heuristic Detection, Log analysis, host-based intrusion detection systems (HIDS), Firewall and Antivirus Software, Manual Incident Response, Disk Imaging and Forensics,Endpoint Encryptions, Policy-based Control.
2011-2015	Behavioural Analysis, Memory Forensics, Network Traffic Analysis, Indicators of Compromise (IoC). Detection, Heuristic Detections, Sandboxing, Anomaly Detection.
2016-2020	Machine Learning and Behaviour Analytics, Endpoint Isolation, Cloud based EDR solutions, File less Malware Detections, Deception Technologies, IoT and OT Endpoint Security, User and Entity Behaviour Analytics (UEBA), (Application and Programming Interface) API Integration and Thread Intelligence Sharing.
2021-2023	Extended Detection and Response (XDR), Zero Trust Architecture, Artificial Intelligence (AI) powered Thread Detection[7][8], Threat Intelligence-driven Defence, Behavioural Biometrics, Cloud-native EDR solutions, Automated Incident response, Ecosystem Integration.

II. Related Work

The article[**2**] « **Investigating Proactive Digital Forensics Leveraging Adversary Emulation** (2022) **)** » of '**Valentine Machaka 1 and Titus Balan**'This literature review defines digital forensics, distinguishes proactive, active, and reactive approaches, and identifies proactive forensics as a means of saving on expenditure and boosting efficiency through preparation of the system for laterinvestigations well in advance. Proactive forensics, the review details, involves the installation of policies, procedures, tools, and technologies for collecting, warehousing, and reviewing evidence ; it also covers related domains such as computerintrusion forensics and forensic readiness in a focus on the use of audit trails and logging ; and it coversintrusion detection technologies (HIDS, NIDS, and EDR) and evidence acquisition use. Also noted are the limitations of the classical forensic approaches in the contemporary system, and the necessity for ahead-of-time preparation for digital forensics, including data integrity and safe communication protection, particularly in the system of the cloud where security and forensics intersect. Proactive digital forensics' conceptual architecture takes on a Security Operations Center (SOC) continuously monitoring endpoints and networks with infrastructure in a state ready for swift incident response and evidence availability. Experimental scenario isolates a social engineering assault with a malicious MicrosoftExcel document containing a Covenant C2 framework payload. On opening up the document and executing the macros, alerts were raised, suggesting a machine-under-the-influence and a suspicious PowerShellfile download request via HTTP. Analysts investigated these suspicions, utilizing tools such as CapME to gather a collection of network traffic data and understand the malicious PowerShellscript block. Such proactive step necessitates verification against sophisticated threats - a gap bridged in later studies. Building on forensic readiness, the work **Fagbohunmi Grifin Siji, Okafor Patrick Uche** proposed[3] this document « **An improved model for comparing different endpoint detection and response tools for mitigating insider threat (**2023)» of The manuscript offers a superior mathematical modeling for determining the relative effectiveness of Endpoint Protection Platforms (EPP) and Endpoint Detection and Response (EDR) solutions in mitigating menace, particularly insider threats. It determines the merits and weaknesses of each platform and provides for selection of the finest protection for organizations of various scales on the basis of specific situations of operation. MATLAB simulation and modeling outcomes demonstrate that EPP is especially effective in loss minimization in cases where threats cause prolonged system inactivity via initial compromise prevention. It is inversely disadvantaged in using machine learning approaches for detection of newly emergent or previously unknown threats but lags behind in tracking all endpoints and promptly responding to threats propagated via the network. Such modeling provides insightful contributions towards customized security strategy consideration for account of menace properties, expenditure, in addition to platform possibilities and acts as a significant contribution towards organizations in security platform selection. yet simultaneously uncover operating shortcomings that are particularly taxing on sophisticated assault vectors**.**This document [4] « **AnEmpirical Assessment of Endpoint Detection and Response Systems against Advanced Persistent Threats Attack Vectors(**2021) **»of «George Karantzas 1 and Constantinos Patsakis »** . The study performs empirical testing of EDRs against APT attacks through a number of various simulated attack situations in controlled networks to observe the performance in such circumstances. It establishes that, despite technological advances, most APT attacks are not deterred or sufficiently reported by the platforms, exposing significant weaknesses in their ability to confront such sophisticated attacks. The paper also examines the various means malicious actors use to evade detection, such as obfuscation and interfering with collection mechanisms, in a way that they are able to breach security controls and move on to more clandestinely operate. Additionally, the study establishes that there exist available telemetry mechanisms, presenting an additional challenge to detection and allowing attacks more difficult to observe. It establishes that, in their current form, most EDRs are not stopping or sufficiently reporting most attacks, and significant upgrades are necessary to make them more effective. Lastly, the study establishes the importance of boosting the detection capabilities, logging and resilience of EDR platforms in an attempt to counteract evasion tactics, and informs future updates and strategic placement of such tools in businesses.This evidence of EDR vulnerabilities against sophisticated attacks necessitates more advanced detection methods, particularly for detecting lateral movement.[5] **« Detecting lateral movement: A systematic survey** »of « **Christos Smiliotopoulosa, Georgios Kambourakisa, Constantinos Kolias(2023)** » It details the increased importance of lateral movement (LM) detection methodologies in today's network security, particularly through the application of Endpoint Detection and Response (EDR) technologies and machine learning technologies, namely neural networks (NN). Several studied works are grounded on log- driven EDR solutions tracking real-time endpoint activity in real-time for identifying anomalous or malicious behavior corresponding to stages of attacks for early and specific detection of privilege escalation or pivoting attacks. Further, deep neural networks (DNN), recurrent models including LSTM, or attention- related models (GAN) are necessary to express and identify complex patterns of anomalous activity in vast amounts of data. Such machine learning technologies extend detection accuracy, especially in IoT or extended IT (IIoT) networks where methods of attacks are more sophisticated and varied. Therefore, the integration of log- driven EDR methodologies and advanced neural network technologies is a promising solution for lateral movement detection challenges, resulting in more reactive and responsive protection mechanisms against sophisticated attacks including APTs or LM.

though the heavy reliance on log analysis introduces new challenges regarding data integrity and manipulation detection.[6] These challenges are directly tackled by **Markus Wurzenberger** et al **« Analysis of statistical properties of variables in log data for advanced anomaly detection in cyber security (2021)** » Variable Type Detector (VTD) introduced in the present work offers a significant contribution towards detecting anomalies in log data, particularly for Endpoint Detection and Response (EDR) solutions. It identifies variable type transitions signifying maliciousity through independent examination of variable portions in logs and thereby boosts the real-time detection capability in EDR solutions. Among the notable achievements of VTD is the autonomous analysis of all the log data without preliminary knowledge or human curation, enabling streamlined implementation in a heterogeneous infrastructure. Moreover, the addition of VTD in currently implemented architectures such as SIEM, EDR, and XDR enriches the level of automation while reducing false positivity, optimizing the functions of remediation and response in cybersecurity. Briefly, VTD offers a solution towards a novel fusion of statistical reasoning and automation towards boosting the efficiency of EDR system in monitoring endpoint log data. Such a feature enables advanced threat detection proactivity and brings security resilience in the infrastructure system as a whole.

Benchmark

To assess the effectiveness of modern incident detection and response (EDR) systems, this study compiles insights from recent research and benchmarking analyses. Current approaches show a wide diversity, including :

- Adversary emulation for proactive forensic investigation
- Dynamic access control models to mitigate insider threats,
- Statistical log analysis and empirical testing

Against advanced persistent threats (APTs). Benchmark results reveal critical tradeoffs : no solution fully covers the MITRE ATT & CK architecture, although several achieve a high level of automation and a low false positive rate. Moreover, there are significant differences in performance overhead, particularly for resource -intensive methods such as machine learning - based behavioral analysis or sandboxing. The need for more portable, understandable, and thorough detection methods is highlighted by these gaps.

Table 2. Comparative cybersecurity detection techniques performance analysis

Article	Technique Focus	Coverage (ATT&CK %)	Performance Impact	False Positives	Detection Precision
Machaka & Balan (2022) – Proactive Forensics	Adversary emulation, DFIR tools	~75%	Moderate	Low	~85%
Siji & Uche (2023) – Insider Threat Mitigation	Mathematical modeling, access mg.	~60%	Moderate-High	Medium	~80%
Karantzas & Patsakis (2021) – EDR vs APT	Sandbox + ML + signature	~65%	High	Medium-High	~70%
Smiliotopoulos et al. (2023) – Lateral Movement Detection	Log analysis + DNN models	~80%	Moderate	Low	~88%
Wurzenberger et al. (2021) – Statistical Log Analysis	Variable Type Detector (VTD)	~78%	Low-Moderate	Low	~90%

III. Anomaly Detection Methods

Malware can be divided into several categories depending on the attackers ' objectives . For example, ransomware encrypts the victim's system to prevent access and demands payment to decrypt files , while spyware is designed to discreetly monitor user actions and steal sensitive information such as passwords or banking details . Cyber criminals are often motivated by the desire to make money, gather industrial or political intelligence , or disrupt the economy and politics . Table 1.1 summarizes the main categories of malware and the most common use of each.

Malware infection vectors are open to modification depending on the targeted operating system. Some popular points of intrusion are PDF files (.pdf), Office suite files such as Word (.doc/.docx), Excel (.xls/.xlsx), PowerPoint (.ppt/.pptx), and even executable files. Executables can exist in many forms : ELF files in Linux-based systems, APK files in Android-based systems, as well as Portable Executables (PE) in Microsoft Windows-based systems. In this study, we focus primarily on the detection and analysis of executable malware samples (e.g., ELF/PE files, implants, RATs) since Windows remains the most widely used operating system worldwide. Besides the type of files used, the attackers utilize an array of attack strategies to achieve a range of malicious objectives such as financial blackmailing, spying, and system

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disruption. These attack forms include phishing attacks, infections with ransomware, denial-of-service (DDoS) attacks, penetrations of networks, and password attacks. Table 1.2 is an explanation of the attacks and matches them with the most common malicious intentions. Effective detection of malware at the endpoint is a high priority in the face of relentlessly evolving cyber threats. The advanced level of the newest attacks underlines the weakness of detection by static signatures alone. A multi-level detection approach—combining advanced static analysis with the predictive capability of machine learning and the consistency of hash comparison—has great promise in bolstering the security of systems. While machine learning has the promise of discovering anomalies and unknown threats previously unknown, static analysis has the ability of discovering suspicious patterns and structures without execution of the actual file itself. Not least of all, the hash-based methods promise rapid detection of recognized kinds of malware. Such a strategic combination aims at significantly enhancing the security posture of Endpoint Detection and Response (EDR) systems against existing and future threats.

Table 3.Attack Types vs Strategic Cyber Threat Objectives

Type of Attack	Financial Extortion	Espionage	Destabilisation
Phishing	✗	✗	✗
Ransomware	✗		
Denial-of-Service (DDoS)			✗
Network Intrusion		✗	✗
Malware (General)	✗	✗	✗
Password Attacks	✗	✗	

IV. Anomaly and Threat Detection Model in Computer Security

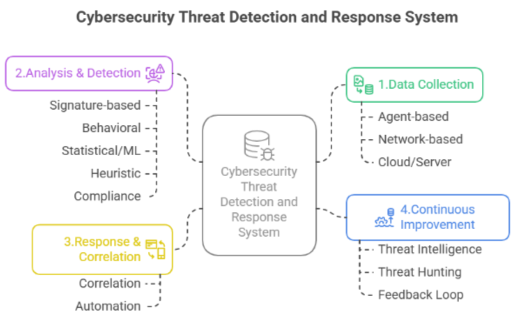

Fig 1. Anomalies detections

Data Collection : The first step is data collection, limited here to endpoint agents and network systems. Windows agents on endpoints collect logs from systems, such as Sysmon or event logs, while Linux agents retrieve kernel traces like eBPF and system activity information. Information is gathered from network devices, including firewalls that capture traffic logs, login attempts, and security alarms. This limited gathering still provides useful visibility by combining perspectives from the host level and the network level, to provide a reliable starting point for centralized correlation and analysis.

Analysis & Detection : In this step , the collected data is analyzed to detect potential threats . Signature-based analysis checks for known patterns using antivirus databases , YARA rules , or IDS rules , allowing malware to be quickly spotted using hashes , domain names, or IP addresses . Behavioral detection focuses on unusual actions like bizarre process chains , persistence attempts, or suspicious use of system tools , often associated with frameworks such as MITRE ATT&CK. Statistical analysis and machine learning add another dimension, allowing for the detection of anomalies , grouping similar behaviors , or classifying files and events based on technical characteristics like headers , imports , or entropy . Heuristics and scoring allow for assigning a risk level to each event to help prioritize the most critical alerts . In our work , we combine static analysis , hashing , and artificial intelligence: static analysis helps extract useful features from files , hashing (including fuzzy hashing) helps identify malware families and variants, and artificial intelligence helps detect threats . unknown or evasive .

Response & Correlation : After detection, the system moves to response and correlation, which aim to understand the attack and stop it quickly . Correlation links events based on time , entity (such as a user, host, or process), or cause, allowing analysts to view the entire attack chain rather than isolated alerts . This reduces noise and groups alerts into meaningful incidents . Automated response via SOAR platforms allows for rapid action , such as isolating an infected endpoint, blocking a checksum , IP address , or domain , containing a malicious email , or resetting compromised accounts . Human analysts remain involved for high - impact decisions, with Tier 1 personnel performing triage and Tier 2 personnel handling escalations . The success of this step is measured by indicators such as mean time to detect (MTTD), mean time to respond (MTTR) and the reduction of false positives.

Continuous Improvement : Since cyber threats are constantly evolving, the system must continuously improve. Every alert and incident provides valuable feedback : false positives and false negatives spotted by analysts are used to retrain machine learning models, adjust thresholds , and refine detection rules . Threat intelligence feeds provide new indicators and techniques used by attackers, which can be used for proactive threat hunting or retrospective research in historical data . Security teams also conduct simulations such as red teaming, purple teaming , and ATT&CK - based exercises to test and validate defenses . Post - incident reviews identify lessons learned, which are then used to improve detection and response capabilities. These continuous improvements ensure that defenses remain effective and able to evolve in the face of new attack techniques.

Feedback Loop : The final step is the feedback loop, which ensures improvements are applied throughout the system. Analysis results, detection effectiveness, and response results are used to update detection playbooks, refine collection strategies, and improve enrichment and machine learning models . Performance metrics like mean time to detect (MTTD), mean time to respond (MTTR), attacker technique coverage, and cost per alert indicate where to focus improvements and demonstrate the overall value of the system.

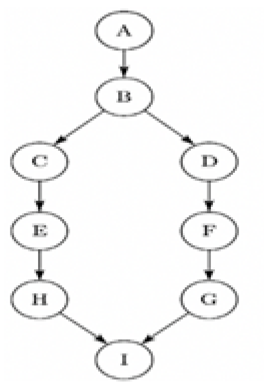

Fig 2. Control flow graph (CFG) G=(V,E) For static analysis

This cycle allows cybersecurity defenses to improve and become more effective over time, providing dynamic and adaptive protection against future cyberattacks.

V. Methodologie

a. Static analysis

Here, we narrow our focus to consider static analysis, which is a form of detailed analysis that entails studying a malware program without running it. Using this method, we are able to look at the internal configuration of the malware, extract its fundamental parts, and ascertain what it might do simply by looking at the artifacts present. Static analysis requires the scrutinizing of metadata, file sections, and pertinent signatures, as well as looking for strange strings, imported functions, and specific commands.

b. Theoretical foundations

Formal theory of language uses Chomsky-type grammars to study languages that are extremely syntactically disciplined. Syntactic investigation, or parsing, checks that code obeys these rules by applying automata and context-sensitive grammars. Finite automata, often built from regular expressions to locate specific patterns, are used to match regular languages, for example in malware detection. Static analysis applies advanced methods such as polyhedral models to compute loop invariants without relying on heuristics. The concise nature of regular expressions allows patterns to be matched in both text and code. Finally, program semantics, independent of syntax, focuses on behavior and meaning.

c. Control Flow Graph (CFG) Definition

A Control Flow Graph **(CFG)** of a program **P** is defined as a directed graph **G(P)=(V,E,entry,exit)** , where V is the set of basic blocks (maximal sequences of instructions without internal jumps) and $E \subseteq V \times V$ represents possible transfers of control. The nodes **entry** and **exit** denote the starting and ending blocks of the program. An edge **(b,b′)∈ E** exists if execution can move from the last instruction of block **b** to the first of block **b′.** A control-flow path is a sequence of connected blocks from **entry** to **exit.** By construction, every concrete program execution corresponds to a path in **G(P),** ensuring that the CFG faithfully represents all possible flows of control in the program.

G=(V,E)

V={v1,v2,...,vn}: set of nodes.

E={(vi,vj)∣ a transition is possible from vi to vj}: set of edges.

Definitions for Data Flow Analysis

Two sets, **Def(n)** and **Use(n),** are defined for every node nnn in the control flow graph **(CFG)** in order to examine data flow. The variables that are defined or assigned a value at node n are contained in the set **Def(n),** whereas the variables that are read or used at that same node are contained in the set **Use(n).** These definitions are essential to several static analysis methods, including def-use chain creation, reaching definitions, and live variable analysis, which all aid in locating potentially harmful activity or redundant and dead code in executable systems.

d. Machine Learning for Malware Classification

The growing number and complexity of malware samples makes traditional analysis methods increasingly challenging. To address this problem. Machine learning offers a scalable and automated method for detecting and classifying malware. By learning patterns and representations specific to malicious behavior, machine learning algorithms can detect structural and behavioral clues from large sets of executable files. This study examines the use of supervised and unsupervised learning techniques to classify malware into known families based on representations of binary and disassembled files.

Dataset and Initial Processing

The dataset used in this work consists of labeled malware samples provided in two complementary formats :

1. **Hexadecimal bytecode representations (. Bytes files)**
2. **Disassembled assembly code (.asm files)**

Each sample is uniquely identified and linked to a specific malware family, as defined in the provided trainLabels.csv file. This file contains manually assigned labels corresponding to nine distinct malware families , numerically coded from 1 to 9. These labels are loaded into a Pandas DataFrame, creating a reference framework for the entire analysis. This organized association between the raw data (bytecode and assembler) and its semantic labels (malware family) is essential to ensure consistency during feature extraction, transformation , and classification . The structure of the dataset thus ensures that each observation is correctly associated with its ground truth throughout the modeling process.

Supervised Learning Models[7]

XGBoost (Extreme Gradient Boosting)

XGBoost is an ensemble learning algorithm based on gradient boosting, which constructs additive decision trees to minimize a loss function augmented by a regularization term. It is particularly suited for structured data and is known for its high performance in classification tasks.

- **Objective function** : $L(\theta)=\sum l(y_i, y'_i) + \sum \Omega(f_k)$
- **Update rule (Newton-Raphson method)** : $W_{t+1}=w_t-(\nabla^2 L)-1\nabla L$
- **Split gain formula** :

$$\text{Gain}=\frac{(\sum gi)2}{\sum hi+\lambda}$$

Where **gi** and **hi** are first and second-order derivatives of the loss function, respectively.

Random Forest

Random Forests are ensemble methods that average predictions from multiple decision trees built on bootstrapped datasets. Each tree uses a randomly selected subset of features, which introduces decorrelation and improves generalization.

- **Splitting criterion**

  Gini index $=1-\sum p_i^2$, where pip_ipi is the proportion of class iii at a node.

- **Feature selection** :

  At each split, only a subset of features is considered, which promotes diversity among trees and stabilizes predictions.

Unsupervised Learning Model[8]

Forest Insulation

While XGBoost and Random Forest are supervised methods , Isolation Forest is an unsupervised algorithm designed specifically for anomaly detection . It isolates anomalies (e.g., new or obfuscated malware) by randomly selecting features and values , building trees in which anomalies are more likely to appear near the root . The main idea is that malicious samples often show behaviors that differ greatly from normal or common malware , making them simpler to identify . This makes Isolation Tree useful for detecting unknown or zero - day malware .

e. Hashing, Fuzzy Hashing, and Hamming Distance in Malware Detection

Une fonction de hachage transforme une entrée x en une sortie de taille fixe h(x) , souvent appelée valeur de hachage ou digest. Les fonctions de hachage cryptographiques classiques, comme MD5 et SHA-256, sont conçues pour être résistantes aux collisions, ce qui signifie qu'il est impossible, du moins de manière computationally viable, de trouver deux entrées différentes x_1 et x_2 telles que $h(x_1) = h(x_2)$. Ces caractéristiques les rendent très utilisés dans les systèmes d'indexation, de vérification de l'intégrité et de détection basée sur les signatures.

In the proposed Endpoint Detection and Response (EDR) architecture, a lightweight agent monitors endpoint activities, collecting information about files , processes, network connections , and system events . Each file is processed by a hashing engine (e.g., **MD5, SHA-256, or TLSH**). This allows for rapid comparison of unknown files with databases containing known malware (blacklists) or verified safe files (whitelists). Suspicious files undergo additional static analysis (e.g. , searching for strings , binary sections , imports) as well as dynamic analysis (e.g. , sandboxing , API call monitoring) . The extracted features are then represented as vectors and sent to machine learning models for classification.

While traditional hashing is efficient , it is binary in nature : two files are either identical (same hash) or completely different (different hash). This limitation makes it unsuitable for combating polymorphic and metabolic malware , as slight code changes result in entirely different hashes . To address this, we introduce fuzzy hashing and Hamming distance as complementary techniques.

Fuzzy hashing [9](e.g., ssdeep, TLSH) generates digests that preserve similarity, allowing for approximate file matching . Instead of requiring exact equality, fuzzy hashes can measure the similarity between two inputs, making them particularly useful against obfuscated or slightly modified malware. **The Hamming distance** [10] indicates how many bits differ between two binary strings of the same length. In malware detection, this can be applied to fuzzy hash outputs or binary representations of extracted signatures. A low Hamming distance means high similarity, suggesting that the new file might be a variant or obfuscated form of a known malware sample.

Why is a low Hamming distance suspicious ?

Malware variants : Attackers often release new versions of malware with minor code modifications . These modifications maintain a high similarity to the original example, resulting in a small Hamming distance.

Obfuscation techniques :[11] Code obfuscation (e.g., reordering instructions , inserting unnecessary code , encrypting code blocks) changes the appearance of the file while maintaining its functionality. Despite these modifications , the obfuscated file remains close to the original in terms of bit-level structure.

Polymorphic malware :[12] More advanced software constantly modifies its code. However, certain structural or behavioral patterns persist, allowing their detection through similarity analysis based on Hamming distance, particularly when focusing on key features rather than entire binaries.

By combining fuzzy hashing and Hamming distance in signature - based detection, the EDR system becomes more resilient to code transformations and malware evolution, thus bridging the gap between exact signature matching and fully behavior - based detection.

Our solution is organized into three main steps : signature matching using fuzzy hashing, static and behavioral feature extraction , and intelligent decision making using machine learning . Unlike traditional cryptographic hashes (like MD5 or SHA-256), fuzzy hashing produces digests that maintain similarity between files , allowing the system to compare files that are not identical but share similar structures . This makes it particularly effective for detecting obfuscated or slightly modified malware . Basic structural similarity can be identified using a low Hamming distance. Poor polymorphic and shapeshifting malware : Although more sophisticated, these types of malware alter their code with each occurrence. However, some parts of the code or behaviors may remain identical, allowing for Hamming distance - based detection, especially if the scan focuses on key features rather than the entire.

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To further refine this process, we incorporate the Hamming distance algorithm to calculate similarity scores between fuzzy hash digests. By comparing the number of bits that differ between a new sample's signature and that of known malware , the system can spot closely related variants or disguised threats.

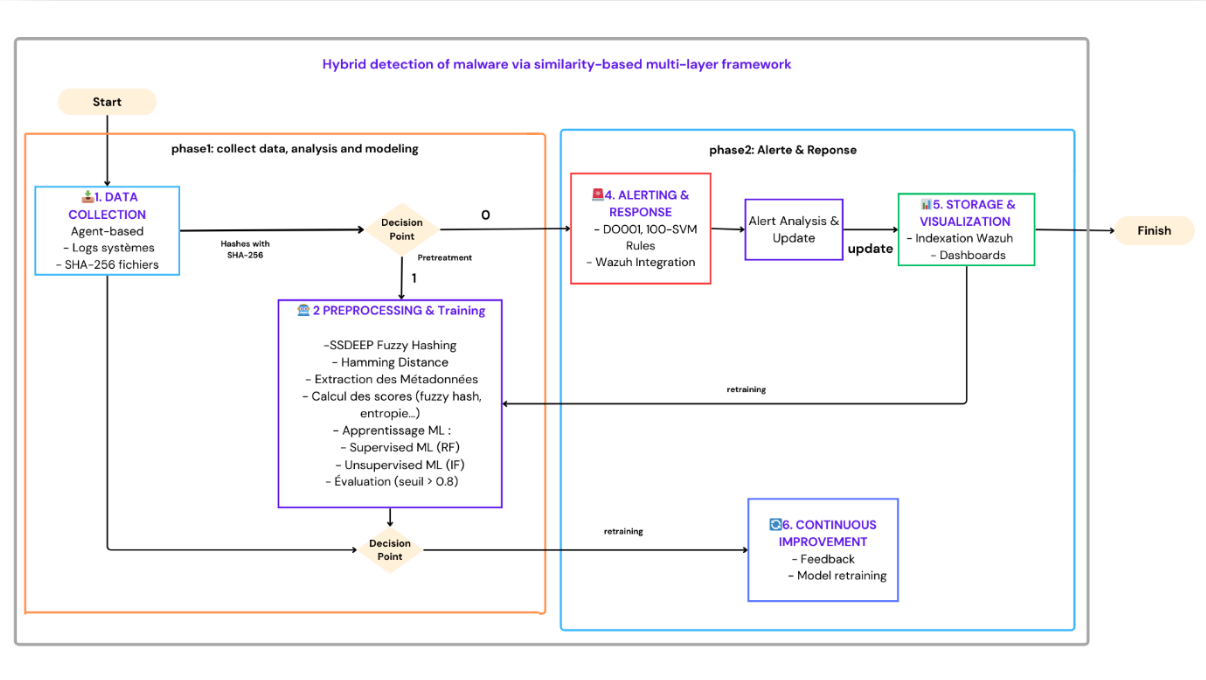

Fig 3.Malware Detection and Analysis Flow

A low Hamming distance indicates a high similarity, suggesting that the file could be a modified version of an existing malware sample.

Phase 1: Collect Data, Analysis, and Modeling

- ***Step 1: Data Collection***

The hybrid detection methodology begins with a comprehensive collection of data for analysis. Agents deployed on target systems capture system logs in real time and calculate **SHA-256**[13] fingerprints of all suspicious files. This first step builds a first line of protection by detecting known files. The system then employs SSDEEP fuzzy hashing to find even partial similarities across files, augmented with Hamming distance calculation and metadata extraction. Before any further processing decisions are taken, these techniques allow for multidimensional examination of dubious files.

- ***Step 2: Preprocessing***

Raw security data must undergo a thorough preparation step before it can be analyzed. By creating comparable signatures for files with common characteristics , SSDEEP fuzzy hashing [14] plays a key role in this situation. This can effectively identify partial similarities that usual cryptographic hashes fail to detect due to minor modifications. By dividing files into blocks , hashing each block , and creating a composite signature for comparison , SSDEEP achieves this . To identify file variations , including modified malware samples , it generates a similarity score between 0 and 100. These drawbacks include difficulties with extremely small files or heavily restructured data , although it works well for code reuse or small modifications . However, SSDEEP is still frequently used in digital forensics , malware analysis, and cybersecurity.

Hamming distance[15] is used to precisely measure the bit - by - bit differences between file signatures, as well as for fuzzy hashing. By comparing their bit patterns, this method is particularly useful for detecting malicious files that have been slightly modified. For example , a Hamming distance of 1 means a close match with known malware when comparing " 101010 " and "101000". This is where this method excels by finding perfect or near- exact matches . Furthermore , by providing important context such as timestamps , file attributes , and behavior patterns , extracting metadata from system logs enhances the study. Data normalization ensures that each feature contributes equally to the subsequent analysis , which helps better prepare the dataset for effective model training . Finally, feature engineering transforms these raw observations into useful variables for machine learning models.

Table 4. Bit Position & Signature

Bit Position	Signature A	Signature B	Difference
1	1	1	0
2	0	0	0
3	1	1	0
4	0	0	0
5	1	0	1
6	0	0	0

Table 5.
Hash-based file classification rules

Condition	Action	Decision Code
SHA-256 or fuzzy hash present in the whitelist	Safe file, no need for ML	0
SHA-256 or fuzzy hash present in the blacklist, or fuzzy hash > 90%	Known malware, immediate alert	0
Fuzzy hash between 20% and 80%	Gray zone → proceed to ML	1
Fuzzy hash < 20%, not present in blacklist or whitelist	Very different → potentially legitimate or unknown → ML can help	1
Fuzzy hash > 80% and < 90%, not present in the blacklist (upper intermediate zone)	ML can help confirm	1

- ***Step 3 : Training the Model***

Both supervised and unsupervised methods are used in the machine learning component. In order to find known threat patterns, Random Forest classifiers learn from labeled malware/benign samples. Isolation Forest algorithms find unusual file properties to uncover new risks. File hashes, information, and behavioral characteristics are among the historical data used to train the models. To maintain detection accuracy, fresh threat intelligence is included into regular retraining cycles. Before models are deployed, performance measurements make sure they fulfill minimum detection thresholds.

Decision Point 1 : Initial Filtering

The system applies initial filtering rules before engaging machine learning. Files matching whitelist entries are immediately cleared as safe. Known malicious files matching blacklist signatures trigger immediate alerts. Files with fuzzy hash similarity scores above 90% to known malware are blocked automatically. Only files in the "gray zone" (20-80% similarity) proceed to ML analysis. This tiered approach optimizes resource usage by reserving ML for ambiguous cases.

Second Decision Point: ML Assessment

Multiple criteria are used by trained models to assess suspicious files. Files with a confidence level greater than 0.8 are marked as malicious and set off response processes. Additional manual evaluation is performed for detections with medium-confidence (0.5-0.8). Files with low confidence are either made public or given more thorough examination. For auditing purposes, all decisions are recorded together with supporting documentation. To increase the accuracy of future detections, the system adds all evaluation results to its knowledge base.

Phase 2: Alerts & Response

The response workflow begins when malware detections are verified. To stop the spread, automated containment procedures isolate impacted systems. Multiple sources of contextual data are used to enhance incident specifics. Prioritization of alerts is determined by the probable impact and the seriousness of the threat. Security teams follow response playbooks as they conduct investigations and corrective actions. Every action is recorded for process improvement and compliance reporting.

- ***Step 4: Alerting & Response***

The alerting system integrates with existing security infrastructure. Wazuh correlation rules trigger notifications based on detection confidence levels. Automated responses may include process termination, file quarantine, or network isolation. Security teams receive enriched alerts with threat intelligence context. Response timelines are tracked to ensure SLA compliance. Feedback loops capture analyst assessments to refine future detections.

- ***Step 5: Visualization & Storage***

A centralized data lake houses all security occurrences. Wazuh indexing makes it possible to quickly search through security data from the past. Threat trends and detection statistics are displayed on custom dashboards. Framework performance measures and areas for improvement are highlighted in reports. Policies for data retention strike a balance between storage limitations and forensic requirements. Appropriate data visibility across teams is ensured by role-based access controls.

- ***Step 6: Continuous Improvement***

Feedback methods are incorporated into the architecture to facilitate continuous improvement. To find gaps in detection, false positives and negatives are examined. Retraining of the model takes place after significant events and at predetermined intervals. Signature databases are updated every day by threat intelligence feeds. Response times and detection rates are monitored using performance metrics. Periodically, the entire system is subjected to security reviews in order to handle new threats.

IV. RESULTS AND DISCUSSION

Precision, Recall, F1-score, and Accuracy are common classification metrics that were used to assess the performance of the implemented models. The global performance for each model is shown in Table 2, while the specific per-class performance (Benign vs. Malware) is summarized in Table 1.

Table 6 .Classification report by class

Model	Class	Precision	Recall	F1-score	Support
XGBoost	0 (Benign)	0.97	0.99	0.98	2282
	1 (Malware)	0.99	0.98	0.99	2998
Random Forest	0 (Benign)	0.97	0.99	0.98	2282
	1 (Malware)	0.99	0.98	0.99	2998
Isolation Forest	0 (Benign)	0.42	0.87	0.56	2282
	1 (Malware)	0.42	0.07	0.13	2998

Table 7. Global model performance

Model	Accuracy	Precision	Recall	F1-score
XGBoost	0.9835	0.9929	0.9780	0.9854
Random Forest	0.9848	0.9949	0.9783	0.9865
Isolation Forest	0.4170	0.4242	0.0747	0.1271

Analysis

The findings show that, with an overall accuracy of over 98%, XGBoost and Random Forest performed almost identically. Both models had F1-scores near 0.99 for both the benign and malware classes, demonstrating exceptional precision (>0.99) and recall (~0.98). This suggests that when trained on characteristics obtained from cryptographic hashes, fuzzy hashing (ssdeep, TLSH), and static analysis attributes, tree-based ensemble approaches are quite successful in detecting malware.

The Isolation Forest model, on the other hand, fared far worse when tested as an unsupervised anomaly detection method. With an F1-score for malware below 0.13 and an overall accuracy of only 41.7%, it was unable to identify the dataset's discriminative patterns. This outcome is in line with predictions since supervised learning techniques are more appropriate when labeled malware and benign samples are available, while isolation forest is better suited for novelty identification in highly imbalanced or unlabeled datasets.

VI. CONCLUSION

All things considered, the tests show that supervised learning models (Random Forest, XGBoost) greatly outperform unsupervised methods like Isolation Forest in static malware detection based on hashing and fuzzy signatures. These results confirm how well machine learning and static analysis work together to provide reliable malware detection.

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The Role Of Influencer Marketing On Consumers Purchase Intentions

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***Abstract*—This study examines the impact of influencer marketing on consumers' purchase intentions in a digital environment increasingly shaped by social media platforms. The research focuses on three key variables: influencer credibility, content relevance, and purchase intention. With the rapid growth of online content creators acting as opinion leaders, particularly among younger generations, understanding the psychological mechanisms through which influencers shape consumer decision-making has become essential.**

A quantitative methodology was adopted using a structured online questionnaire administered to 104 active social media users in Morocco. The collected data were analyzed using SPSS, employing statistical techniques such as Spearman correlation and linear regression to evaluate the relationships between the studied variables.

The results reveal that influencer credibility and content relevance significantly enhance the perceived effectiveness of influencer marketing, which in turn positively influences consumers' purchase intentions. The findings indicate that the more credible and relatable an influencer is perceived to be, and the more relevant the shared content is, the higher the likelihood that consumers will consider purchasing the promoted products or services.

This research provides both theoretical and managerial insights by highlighting the importance of trust, authenticity, and content alignment in digital communication strategies. It also offers practical recommendations for brands seeking to optimize their influencer marketing campaigns and improve return on investment in a competitive digital marketplace.

***Index Terms*—Influencer Marketing, Influencer Credibility, Purchase Intention, Content Relevance, Consumer Behavior, SPSS**

I. Introduction

The emergence of social media has profoundly transformed the communication dynamics between brands and consumers. In this rapidly evolving digital ecosystem, influencers—such as bloggers and content creators on platforms like Instagram, YouTube, and TikTok—have become central actors in modern marketing strategies. These individuals, often followed by large online communities, possess the ability to recommend, promote, and evaluate products in ways that are perceived as more authentic and relatable than traditional advertising. Consequently, influencer marketing has emerged as a strategic tool that enables brands to shape consumer attitudes, perceptions, and ultimately purchasing decisions.

The growing importance of influencer marketing is closely linked to changes in consumer behavior, particularly among younger generations who increasingly rely on online recommendations and peer opinions. In a context where consumers are more informed and often skeptical of traditional advertising, companies are seeking to establish more personalized and trust-based relationships with their audiences. Influencer marketing thus appears as a powerful lever to enhance engagement, build trust, and stimulate purchase intention.

However, despite its rapid expansion and widespread adoption by firms, the actual effectiveness of influencer marketing remains insufficiently quantified and theoretically explored. Questions persist regarding the psychological and social mechanisms through which influencers affect consumer decision-making, as well as the key factors that determine the success of influencer campaigns. In particular, elements such as perceived influencer credibility, authenticity of communication, and relevance of shared content may play a decisive role in shaping consumer responses.

In this context, the present study aims to better understand how influencer marketing influences consumers' purchase intentions and to identify the main determinants of its effectiveness. More specifically, this research seeks to analyze the impact of influencer credibility and content relevance on the perception of influencer marketing and, subsequently, on consumers' purchase intentions. By adopting a quantitative approach based on survey data collected from active social media users in Morocco, this study contributes to a deeper understanding of digital influence mechanisms and provides empirical evidence on the effectiveness of influencer marketing as a contemporary communication strategy.

II. Motivation & Methodology

A. Key Concepts

Influencer Marketing: According to Kapitan and Silvera (2016), influencer marketing is based on the power of an individual to shape the perceptions, attitudes, and intentions of an audience through digital media, particularly social media platforms. This approach relies on mechanisms of social persuasion, where influencers play a central role due to their perceived authenticity and proximity to their followers. This definition highlights the emotional and behavioral bond between the opinion leader and their community. Influence is therefore not solely based on notoriety, but also on trust, frequency of interaction, and the perceived value of the information delivered. In this context, the influencer becomes a key reference figure and a “model of consumption” for promoted products or services.

According to Uzunoğlu and Kip (2014), influencer marketing can be conceptualized as a process in which an individual’s network, reputation, and digital authority are leveraged by brands to relay commercial messages and enhance brand image. This socio-technical perspective suggests that influence emerges from the structural characteristics of digital platforms, such as algorithms, visibility, and content virality. Consequently, the performance of an influencer campaign depends not only on the influencer or the message itself, but also on the topology of their network and the design of the social media environment.

Purchase Intention: For Kotler and Keller (2016), purchase intention refers to a consumer’s conscious predisposition to buy a product or service in the near future. This concept emphasizes that intention is not a spontaneous action but the result of a rational and psychological decision-making process involving the evaluation of expected benefits, comparison of available alternatives, and preparation for action. Thus, purchase intention represents a crucial stage between a favorable attitude toward a product and the actual purchase decision.

Fishbein and Ajzen (1975) define purchase intention as the willingness or disposition to perform a specific behavior, in this case the act of purchasing. It is considered one of the best predictors of actual behavior, as it reflects the individual’s motivation and mental commitment to act. Their theoretical framework underlines the role of beliefs and attitudes, where beliefs about product attributes shape attitudes, which in turn influence purchase intention. Therefore, intention is a strong psychological construct that largely determines whether purchasing behavior will occur.

Ajzen (1991), through the Theory of Planned Behavior, provides a more comprehensive perspective by integrating several determinants of intention. According to this theory, purchase intention results from the interaction of three main factors:

- *Attitude toward behavior:* the positive or negative evaluation made by the consumer regarding the purchase.
- *Subjective norms:* the perceived social pressure from relatives or society to perform or not perform the purchase.
- *Perceived behavioral control:* the perceived ease or difficulty of performing the purchase, including both external factors (availability, budget) and internal factors (trust, competence).

Purchase intention can thus be viewed as an integrated measure of motivation to act, taking into account personal preferences, social context, and perceived control over the behavior.

Consumer Behavior: Schiffman and Kanuk (2010) describe consumer behavior as the processes through which consumers search for, purchase, use, evaluate, and dispose of products and services that satisfy their needs. From a broader perspective, Doyle (2016) defines consumer behavior as a decision-making process carried out by identifiable groups of consumers when selecting products or services. This process is influenced by consumer attitudes as well as contextual factors such as time, place, and price.

Furthermore, Solomon (2018) states that consumer behavior encompasses all the processes involved when individuals or groups select, purchase, use, or dispose of products, services, ideas, or experiences in order to satisfy their needs and expectations. Understanding consumer behavior is therefore essential for developing effective marketing strategies, as it enables firms to adapt their products, messages, and communication channels to customer expectations and evolving market demands.

B. Conceptual Model

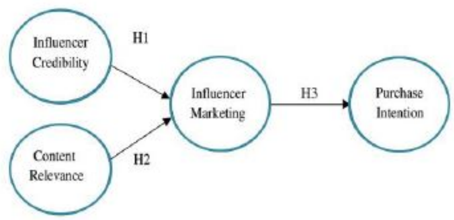

Fig. 1. Presentation of the conceptual model

C. Conceptual Model and Theoretical Framework

The conceptual model adopted in this study is based on three main hypotheses establishing relationships between independent variables, a mediating variable, and a dependent variable. These variables are defined as follows.

Independent Variables:

Influencer Credibility: Influencer credibility refers to the extent to which the audience perceives the influencer as trustworthy, expert, and attractive. A credible source exerts a stronger influence on consumer attitudes and behaviors (Ohanian, 1990). This credibility dimension plays a fundamental role in strengthening persuasion in marketing communications, particularly in digital environments where consumers rely heavily on perceived authenticity and trust.

Content Relevance: Content relevance refers to the degree of alignment between the message delivered by the influencer and the expectations, needs, and interests of the target audience. Relevant and engaging content enhances consumer attention and encourages a more favorable perception of the brand (De Veirman et al., 2017). Therefore, content relevance can be considered a key factor that conditions consumer engagement and positive behavioral responses.

Mediating Variable:

Influencer Marketing: Acting as an intermediary variable, influencer marketing represents the process through which influencers transmit promotional messages that shape consumer perceptions (Lou & Yuan, 2019). It functions as a relational mechanism that transforms the impact of independent variables, namely credibility and content relevance, into consumer attitudes and behavioral intentions. In this sense, it serves as a mediator between message exposure and purchase intention.

Dependent Variable:

Purchase Intention: Purchase intention corresponds to the consumer's expressed willingness to acquire a specific product or service (Dodds, Monroe & Grewal, 1991). In marketing research, it is considered a crucial predictive variable, as it helps anticipate future purchasing behavior. While the actual purchase is an observable action, intention is a psychological construct that strongly predicts behavioral outcomes.

1) Theoretical Justification of the Model: To explain how influencers affect purchase intentions, the proposed model relies on several well-established theoretical foundations. First, the Theory of Planned Behavior (Ajzen, 1991) suggests that behavioral intention is one of the strongest predictors of actual behavior. In this context, when a consumer develops an intention to purchase, it is often the result of prior psychological and social influences, including exposure to persuasive communication such as influencer marketing.

Furthermore, classical studies by Hovland and Weiss (1951) demonstrate that source credibility significantly enhances the persuasive effectiveness of a message. When an influencer is perceived as competent, honest, and attractive, their recommendations are more likely to generate trust and stimulate purchase intention. This justifies the integration of influencer credibility as a key independent variable in the conceptual model.

In addition, the quality and relevance of the content shared by influencers play a central role in shaping consumer perceptions. According to Kapferer (2008), messages that are perceived as relevant and engaging foster stronger identification with the brand and positively influence consumer behavior. In this framework, influencer marketing acts as a linking mechanism between message characteristics and the final purchasing decision, reinforcing its role as a mediating variable.

Moreover, recent research, particularly that of Lou and Yuan (2019), highlights that the combined effect of influencer credibility and content relevance contributes to building consumer trust and enhancing purchase intention. This interaction justifies the structural relationships proposed in the conceptual model.

2) Formulation of Research Hypotheses: To empirically test the relationships between the variables of the conceptual model, three main hypotheses were formulated based on the literature in marketing, persuasive communication, and consumer behavior:

- **H1:** Influencer credibility positively impacts the perception of influencer marketing. Influencer credibility enhances consumer trust in promotional messages. When influencers are perceived as authentic, reliable, and expert, their content is more likely to be accepted, thereby strengthening the effectiveness of influencer marketing strategies.
- **H2:** Content relevance positively influences influencer marketing. This hypothesis is grounded in studies on message relevance and consumer engage-

ment (Kapferer, 2008; De Veirman et al., 2017). Content that is useful, informative, and aligned with consumer expectations improves message reception and increases the perceived value and effectiveness of influencer marketing.

- **H3:** Influencer marketing positively impacts purchase intention. Based on the Theory of Planned Behavior (Ajzen, 1991), the perception of an effective persuasive lever, such as influencer marketing, directly influences the intention to act, namely the purchase decision. Consequently, consumers who perceive influencer campaigns as convincing are more likely to consider purchasing the promoted products or services.

III. Methodology

A. Type of Research and Methodological Strategy

1) Research Type: This study adopts a quantitative research approach, as it enables an accurate measurement of consumer perceptions and behaviors toward influencer marketing. The primary objective is to collect numerical data in order to statistically analyze the relationships between the variables of the conceptual model, namely influencer credibility, content relevance, and purchase intention.

By using a quantitative methodology, the research aims to empirically validate the theoretical relationships proposed in the conceptual framework while providing concrete and actionable results for both academic research and digital marketing practitioners. This approach goes beyond subjective impressions by offering a structured, objective, and reliable interpretation of consumer behavior in the context of influencer marketing.

2) Epistemological Posture: The epistemological posture adopted in this study is positivist, which is based on the principle that knowledge should rely on objective, measurable, and verifiable facts. From this perspective, social phenomena—such as the impact of influencer marketing on purchase intention—can be understood through observable and measurable causal relationships.

Through the use of quantitative methods and statistical testing, the study seeks to collect factual data and analyze the relationships between variables in a scientific and objective manner. The positivist paradigm thus ensures the reliability, rigor, and generalizability of the findings derived from empirical evidence.

3) Research Strategy Adopted: To achieve the research objectives, a survey-based strategy was employed using a structured online questionnaire. This strategy follows a hypothetico-deductive approach, which is fully aligned with the quantitative nature of the study. It allows for the collection of a significant volume of structured data, facilitating hypothesis testing and statistical validation of the conceptual model.

B. Data Collection

1) Data Collection Tool: For this study, Google Forms was selected as the primary data collection instrument. This online platform offers several advantages that align with the requirements of quantitative research. It enables the rapid design, distribution, and management of questionnaires in an accessible and efficient manner for both researchers and respondents.

The questionnaire was structured according to the conceptual model and included various types of questions such as Likert-scale items, multiple-choice questions, and closed-ended responses. Google Forms also allows real-time response tracking and easy export of data to statistical software, which is essential for rigorous quantitative analysis. Furthermore, the platform is free, widely accessible via a simple link, and compatible with multiple digital devices (smartphones, tablets, and computers), thereby maximizing the response rate.

2) Sampling:

a) Definition of the Target Population: The target population of this study consists of active social media users in Morocco. According to the Digital 2025 Morocco report (DataReportal), approximately 21.3 million Moroccans use social media, representing about 55.5% of the total population. This highlights the growing importance of social media as a key channel for communication, information, and commerce in the Moroccan context.

Users of these platforms are particularly exposed to influencer marketing strategies, especially young adults who represent a large proportion of social media users. Platforms such as Facebook, Instagram, TikTok, and YouTube serve as primary channels of interaction between influencers and consumers, significantly shaping purchasing behaviors. Therefore, this population is highly relevant for analyzing the impact of influencer marketing on purchase intentions.

By focusing on active social media users, the study aims to better understand how individuals perceive influencers, how they react to promotional content, and to what extent these interactions influence their purchasing intentions. This population is relevant both academically, for understanding evolving consumer behavior, and managerially, for guiding digital marketing strategies.

b) Sampling Method: A probability sampling technique, specifically simple random sampling, was employed in this study. This method ensures that each member of the target population has an equal probability of being selected to participate in the survey, thereby reducing selection bias and enhancing the representativeness of the sample. The use of simple random sampling is justified by the objective of the study, which is to objectively and reliably assess the impact of influencer marketing on consumer purchase intention. The questionnaire was distributed online through social media platforms such as WhatsApp, Instagram, and Facebook, allowing for rapid and efficient data collection from diverse respondents.

c) Sample Size: The final sample consisted of 104 valid responses, collected with an estimated margin of error of 10% and a confidence level of 90%. This sample size is considered adequate to obtain representative and usable results in line with the research objectives.

Applying the sample size estimation formula to the population of social media users in Morocco, and assuming a 10% margin of error with a 90% confidence level, an approximate minimum sample size of 69 respondents was deemed sufficient. The obtained sample size of 104 responses therefore exceeds the minimum requirement, ensuring a reliable estimation of consumer purchase intentions while maintaining a balance between feasibility and statistical accuracy.

3) Data Analysis Method: For data analysis, the Statistical Package for the Social Sciences (SPSS) software was utilized. SPSS is widely recognized for its reliability and robustness in processing quantitative survey data. It enables the execution of various statistical analyses, including descriptive statistics, correlation analysis, regression analysis, and reliability testing.

The selection of SPSS was motivated by its capacity to organize, process, and interpret data rigorously through an accessible and scientifically validated interface. This methodological choice ensures a thorough and systematic analysis that meets the scientific standards required for empirical research.

4) Descriptive Analysis: **Influencer Credibility.** Influencer credibility, measured through questions related to honesty and trust in influencers' opinions, is perceived as moderate by respondents. For Question 6 ("Do you think the influencers you follow are honest in their recommendations?"), the mean score is 2.53 with a standard deviation of 1.061, indicating a relatively mixed perception regarding influencer honesty. Similarly, Question 7 ("Do you trust the opinions shared by the influencers you follow?") shows a mean of 2.48 and a standard deviation of 1.149, suggesting that while some respondents express trust in influencers, a considerable proportion remains hesitant or skeptical. These results indicate that influencer credibility is present but not strongly established, highlighting the need for greater transparency, authenticity, and consistency in influencer communication to enhance perceived credibility.

Content Relevance. The content published by influencers is generally perceived as informative and relevant. Question 8 ("Do you find the content published by the influencers you follow informative?") has a mean of 3.06 and a standard deviation of 1.122, indicating that respondents tend to consider influencer content useful for understanding products. Regarding Question 9 ("Do influencer posts catch your attention?"), the mean score is 2.99 with a standard deviation of 1.145, reflecting a moderate level of attention attracted by influencer posts. Furthermore, Question 10 ("Does the content proposed by influencers help you better understand the products?") records the highest mean (3.21) with a standard deviation of 1.196, suggesting that influencer content plays a significant informational role in the consumer decision-making process. Overall, these findings emphasize that relevant and informative content contributes positively to consumer engagement and enhances the perceived effectiveness of influencer communication.

Influencer Marketing Impact. The results highlight the tangible influence of influencer marketing on consumer behavior. Question 11 ("Do influencer posts affect your opinion about certain brands?") presents a mean of 3.01 and a standard deviation of 1.145, indicating that influencer communication moderately shapes brand perceptions. Additionally, Question 12 ("Do you remember products you have seen promoted by influencers?") shows a mean of 2.86 and a standard deviation of 1.018, suggesting that influencer-promoted products remain relatively memorable among respondents. These findings confirm that influencer marketing contributes to brand awareness and cognitive engagement, reinforcing its strategic importance in digital marketing.

Purchase Intention. Purchase intention appears to be influenced by influencer recommendations, although it remains moderate overall. Question 13 ("Have you ever bought a product after seeing it recommended by influencers?") has a mean of 1.44 (coded as Yes = 1 and No = 2), indicating that the majority of respondents have already made a purchase following an influencer's recommendation. This result demonstrates the concrete impact of influencer marketing on actual consumer behavior. However, Question 14 ("Are you willing to buy a product recommended by one or more influencers?") records a mean of 2.83 with a standard deviation of 1.153, suggesting that purchase intention is present but not particularly strong. This implies that while consumers are receptive to influencer-promoted products, this interest does not always translate into immediate purchasing actions. Therefore, additional marketing strategies such as promotional incentives, discounts, or trust-building

TABLE I
DESCRIPTIVE STATISTICS OF SURVEY ITEMS

Item	N	Min	Max	Mean	Std. Dev.
6. Do you think the influencers you follow are honest in their recommendations?	104	1	5	2.53	1.061
7. Do you trust the reviews shared by the influencers you follow?	104	1	5	2.48	1.149
8. Do you find the content published by the influencers you follow to be informative?	104	1	5	3.06	1.122
9. Do influencer posts catch your attention?	104	1	5	2.99	1.145
10. Does the content proposed by influencers help you better understand the products?	104	1	5	3.21	1.196
11. Do influencer posts affect your opinion about certain brands?	104	1	5	3.01	1.145
12. Do you ever remember products you have seen promoted by influencers?	104	1	5	2.86	1.018
13. Have you ever bought a product after seeing it recommended by influencers?	104	1	2	1.44	0.499
14. Are you willing to buy a product recommended by one or more influencers?	104	1	5	2.83	1.153
Valid N (listwise)			104		

mechanisms may be necessary to convert intention into actual purchase behavior.

TABLE II
RELIABILITY OF MEASUREMENT SCALES (CRONBACH'S ALPHA)

Variable	Cronbach's Alpha (α)	Reliability Level
Influencer Credibility	0.858	Very Good
Content Relevance	0.781	Good
Influencer Marketing	0.872	Excellent

5) Reliability Analysis: Source: Authors' elaboration based on SPSS outputs.

The reliability of the measurement scales was assessed using Cronbach's Alpha coefficient. As shown in Table II, all alpha values exceed the recommended threshold of 0.7, indicating satisfactory internal consistency of the scales.

More specifically, the Influencer Credibility scale presents a Cronbach's Alpha of 0.858, reflecting very good reliability, while the Content Relevance scale records an alpha of 0.781, indicating good reliability. The Influencer Marketing scale shows the highest coefficient (0.872), demonstrating excellent internal consistency among its items.

Overall, the Cronbach's Alpha values ranging between 0.781 and 0.872 confirm that the items within each construct are coherent and reliably measure their respective dimensions. These results validate the internal reliability of the measurement instruments and support the robustness and consistency of the data for subsequent statistical analyses.

6) Interpretation of Reliability Results: **Influencer Credibility.** The Cronbach's Alpha value of 0.858 for the influencer credibility scale indicates excellent internal consistency. This result suggests that the two items used to measure influencer credibility, namely the honesty of recommendations and the trust in shared opinions, are strongly correlated and consistently capture the same underlying construct of perceived credibility. A high alpha coefficient reflects the robustness of the scale and confirms that respondents provided highly consistent answers across these items. Consequently, the scale can be considered reliable for assessing individuals' perceptions of the trustworthiness and authenticity of influencer recommendations.

Content Relevance. For the content relevance scale, the Cronbach's Alpha coefficient is 0.781, which indicates good reliability. This value shows that the three items related to informativeness, attention attraction, and the ability of influencer content to help consumers better understand products are reasonably well correlated. Although the alpha is slightly lower than that of the credibility scale, it remains above the recommended threshold of 0.7, confirming the adequacy of the scale. The slight variation may reflect differences in how respondents perceive various aspects of content relevance; however, the overall consistency remains satisfactory. These findings indicate that participants generally share similar perceptions regarding the usefulness and relevance of influencer-generated content.

Influencer Marketing. The influencer marketing scale records a Cronbach's Alpha of 0.872, indicating excellent internal reliability. This high coefficient demonstrates that the items measuring the impact of influencer posts on brand opinion and product recall are highly consistent in capturing the effectiveness of influencer marketing.

7) Correlation Analysis: To examine the relationships between the variables included in the conceptual model, a Spearman rank correlation test was employed. This non-parametric method is particularly suitable for ordinal data and for situations where the assumptions of normality are not fully satisfied. The choice of Spearman's correlation is justified by the nature of the collected data, which were measured using 5-point Likert scales and are therefore considered ordinal.

Moreover, preliminary descriptive analyses revealed slightly asymmetric distributions, further supporting the use of a non-parametric approach rather than Pearson's correlation, which requires stronger assumptions regarding normality and linearity. In contrast, the Spearman test is more robust to deviations from normality and is appropriate for assessing monotonic relationships between variables.

The Spearman correlation coefficient allows the measurement of both the strength and the direction of the association between two constructs. In this study, it was used to analyze the relationships between influencer credibility, content relevance, influencer marketing, and purchase intention.

The results indicate the presence of significant positive correlations among all the main variables of the model. In particular, influencer credibility and content relevance are positively associated with influencer marketing, which in turn shows a positive relationship with purchase intention. These findings confirm the structural coherence of the conceptual model and provide empirical support for the research hypotheses formulated in this study.

TABLE III
SPEARMAN CORRELATION BETWEEN CONTENT RELEVANCE AND INFLUENCER MARKETING

Spearman's Rho	Content Relevance	Influ Marketing
Content Relevance	1.000	0.694**
Sig. (2-tailed)	–	0.000
N	104	104
Influ Marketing	0.694**	1.000
Sig. (2-tailed)	0.000	–
N	104	104

** Correlation is significant at the 0.01 level (2-tailed).
Source: Spearman analysis performed using SPSS.

8) Interpretation of Spearman Correlation Results: The Spearman correlation coefficient between influencer credibility and the perception of influencer marketing is $\rho = 0.694$. This value indicates a strong and positive correlation between the two variables. In other words, the higher the perceived credibility of the influencer, the more favorable the perception of influencer marketing among respondents.

Furthermore, the associated p-value is equal to 0.000 ($p < 0.01$), which confirms that the correlation is statistically significant at the 1% significance level. This result implies that the probability of observing such a relationship by chance is extremely low, thereby supporting the robustness of the statistical findings.

From a theoretical perspective, these results highlight the central role of influencer credibility in shaping consumers' perceptions of influencer marketing. When an influencer is perceived as trustworthy, authentic, and competent, consumers are more likely to positively evaluate the marketing messages they convey. Consequently, credibility appears to be a key determinant in enhancing the effectiveness of influencer marketing strategies and reinforcing consumer receptiveness to promotional content.

TABLE IV
SPEARMAN CORRELATION: CONTENT RELEVANCE AND INFLUENCER MARKETING

Spearman's Rho	Content Relevance	Influencer Marketing
Content Relevance	1.000	0.570**
Sig. (2-tailed)	–	0.000
N	104	104
Influencer Marketing	0.570**	1.000
Sig. (2-tailed)	0.000	–
N	104	104

** Correlation is significant at the 0.01 level (2-tailed).
Source: Spearman analysis performed using SPSS.

H2 : Content relevance positively impacts the perception of influencer marketing Spearman correlation coefficient : 0.570 A coefficient of 0.570 indicates a moderate to strong correlation between content relevance and perception of influencer marketing.This suggests that relevant content has a direct and significant effect on how consumers perceive influencer marketing campaigns. p-value : 0.000 The p-value is also less than 0.01 (p ¡ 0.01), indicating that this correlation is statistically significant. Interpretation : Content relevance is a key factor in consumers' positive perceptions of influencer marketing. When content is deemed relevant, it promotes better reception of influencer marketing campaigns.

TABLE V
SPEARMAN CORRELATION: INFLUENCER MARKETING AND PURCHASE INTENTION

Spearman's Rho	Influ Marketing	Purchase Intention
Influ Marketing	1.000	0.765**
Sig. (2-tailed)	–	0.000
N	104	104
Purchase Intention	0.765**	1.000
Sig. (2-tailed)	0.000	–
N	104	104

** Correlation is significant at the 0.01 level (2-tailed).
Source: Spearman analysis performed using SPSS.

H3: A positive perception of influencer marketing positively impacts purchase intention.

The Spearman correlation coefficient between influencer marketing and purchase intention is $\rho = 0.765$. This value indicates a very strong positive correlation between the two variables. In other words, the more positively consumers perceive influencer marketing, the higher their intention to purchase the promoted products.

Furthermore, the associated p-value is equal to 0.000 ($p < 0.01$), which confirms that the relationship is statistically significant at the 1% significance level. This demonstrates that the observed association is not due to chance and reflects a robust statistical relationship between the constructs.

From a theoretical and managerial perspective, these findings suggest that influencer marketing constitutes a powerful driver of consumer purchase intention. When influencer campaigns are perceived as credible, engaging, and persuasive, they significantly enhance consumers' willingness to consider and purchase recommended products. Therefore, strengthening the effectiveness and authenticity of influencer marketing strategies can directly contribute to increasing purchase intention among consumers.

9) Linear Regression Analysis: Variables Introduced in the Regression Model :

TABLE VI
VARIABLES ENTERED IN THE REGRESSION ANALYSIS

Model	Variables Entered	Variables Removed	Method
1	Influencer Marketing	None	Enter

Dependent Variable: Purchase Intention.
All requested variables were entered.
Source: Authors' elaboration based on SPSS output.

The results presented in Table VI confirm that the independent variable, namely influencer marketing, was successfully introduced into the regression model using the enter method. The dependent variable considered in this analysis is purchase intention. This modeling choice is consistent with the conceptual framework, where influencer marketing is assumed to directly influence consumers' purchase intention.

TABLE VII
SUMMARY OF REGRESSION MODEL

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	0.774	0.599	0.596	0.37821

Predictors: (Constant), Influencer Marketing.
Source: Authors' elaboration based on SPSS output.

The model summary indicates a strong relationship between influencer marketing and purchase intention. The correlation coefficient ($R = 0.774$) reflects a strong positive association between the predictor and the dependent variable.

Moreover, the coefficient of determination ($R^2 = 0.599$) shows that approximately 59.9% of the variance in purchase intention is explained by influencer marketing. The adjusted R^2 value (0.596) confirms the robustness and stability of the model, indicating that the explanatory power remains high even after adjustment.

Additionally, the standard error of the estimate (0.37821) suggests a relatively low dispersion of the observed values around the regression line, which reflects a good predictive accuracy of the model. Overall, these results demonstrate that influencer marketing is a significant predictor of purchase intention, thereby empirically supporting hypothesis H3.

TABLE VIII
OVERALL TEST OF SIGNIFICANCE OF THE REGRESSION MODEL (ANOVA)

Model	Sum of Squares	df	Mean Square	F	Sig.
Regression	21.785	1	21.785	152.294	0.000[b]
Residual	14.590	102	0.143	–	–
Total	36.375	103	–	–	–

[a] Dependent Variable: Purchase Intention.
[b] Predictors: (Constant), Influencer Marketing.
Source: Authors' elaboration based on SPSS output.

The ANOVA results presented in Table VIII confirm the overall statistical significance of the regression model. The F-statistic value ($F = 152.294$) is highly significant with a p-value equal to 0.000 ($p < 0.01$), indicating that the model provides a significantly better fit than a model with no predictors.

This result demonstrates that influencer marketing has a statistically significant effect on purchase intention. In other words, the independent variable included in

the model contributes meaningfully to explaining the variability in the dependent variable.
Furthermore, the relatively high regression sum of squares (21.785) compared to the residual sum of squares (14.590) suggests that a substantial proportion of the variance in purchase intention is explained by influencer marketing. Overall, the ANOVA test validates the robustness and explanatory power of the regression model, thereby confirming the relevance of the hypothesized relationship between influencer marketing and purchase intention.

TABLE IX
REGRESSION COEFFICIENTS FOR THE MODEL

Model	B	Std. Error	Beta	t
(Constant)	0.426	0.162	–	2.623
Influencer Marketing	0.800	0.065	0.774	12.3410

Dependent Variable: Purchase Intention.
Source: Authors' elaboration based on SPSS output.

10) Interpretation of Regression Coefficients: The regression results indicate that influencer marketing has a significant and positive impact on purchase intention. The unstandardized coefficient ($B = 0.800$) shows that a one-unit increase in the perception of influencer marketing leads to a corresponding increase of 0.800 units in purchase intention.

Furthermore, the standardized coefficient ($\beta = 0.774$) confirms the strong explanatory power of this variable, highlighting its substantial influence on consumers' behavioral intentions. The high t-value ($t = 12.341$) combined with a p-value of 0.000 ($p < 0.01$) demonstrates that the effect is highly statistically significant. These findings confirm that influencer marketing is a key determinant of consumers' purchase intention and plays a central role in shaping purchasing behavior in digital environments.

11) Summary of Results: The empirical analysis allowed the validation of all the research hypotheses formulated in the conceptual model:

- **H1: Validated** – Influencer credibility positively impacts the perception of influencer marketing.
- **H2: Validated** – Content relevance positively impacts the perception of influencer marketing.
- **H3: Validated** – A positive perception of influencer marketing positively influences purchase intention.

Overall, the results reveal positive and statistically significant relationships between all the variables included in the proposed model. These findings suggest that the effectiveness of influencer marketing campaigns largely depends on the credibility of the influencer and the relevance of the content shared. Enhancing these two dimensions leads to a more favorable perception of influencer marketing, which in turn increases consumers' purchase intentions.

12) Managerial and Theoretical Implications: From a managerial perspective, the findings provide practical insights for digital marketing professionals and companies seeking to collaborate with influencers. The results highlight that both influencer credibility and content relevance play a decisive role in the consumer decision-making process. Therefore, marketing managers should carefully select influencers based on their authenticity, perceived expertise, and the level of trust they generate among their audience. In addition, particular attention should be paid to the quality, coherence, and relevance of the disseminated content in order to maximize the effectiveness of influencer marketing campaigns (Jin & Phua, 2014). Integrating these elements into the overall communication strategy can significantly enhance campaign performance and stimulate purchase intention.

From a theoretical perspective, this study contributes to the existing literature on influencer marketing by empirically supporting key assumptions derived from the Elaboration Likelihood Model (ELM), particularly regarding central and peripheral routes of information processing (Petty & Cacioppo, 1986). The results suggest that when influencer content is perceived as relevant and the source as credible, consumers are more likely to engage in deeper cognitive processing, thereby strengthening persuasive outcomes in digital contexts.

Furthermore, this research underscores the importance of psychological and perceptual variables in shaping message reception and consumer behavior. It opens new avenues for future research, especially by examining additional moderating variables, cross-cultural differences, or sector-specific consumer responses to influencer marketing strategies (Lou & Yuan, 2019).
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IV. CONCLUSION

This research was conducted in a context where digital marketing, and more specifically influencer marketing, occupies a central role in contemporary brand strategies. The study examined the impact of influencer marketing on consumer purchasing behavior by focusing on three key variables: influencer credibility, content relevance, and purchase intention.

By adopting a quantitative methodology and using a structured questionnaire distributed via Google Forms, data were collected from a sample of 110 respondents. The collected data were analyzed using SPSS software, which enabled the identification of significant correlations and causal relationships between the studied variables. The findings reveal that higher perceived influencer credibility and greater content relevance significantly enhance consumers' purchase intention. This dynamic underlines the strategic importance of influencers in the modern consumer decision-making process.

The main objective of this research was to better understand how influencer marketing, as a digital communication strategy, influences consumer purchasing decisions. The empirical results provided clear and measurable answers to the research problem. On the one hand, the study confirmed the critical role of perceived influencer credibility. Respondents who perceive influencers as honest, competent, and trustworthy are more likely to consider their recommendations when making purchasing decisions. On the other hand, content relevance emerged as a key determinant in this process, as useful, informative, and well-targeted content positively shapes consumer attitudes and strengthens purchase intention.

Thus, the research successfully achieved its objectives by clarifying the relationships between the variables of the proposed conceptual model and by providing a solid empirical foundation for future applications in digital marketing research.

From a theoretical perspective, this study contributes to the existing literature on influencer marketing by confirming the central role of influencer credibility and content quality in stimulating purchase intention. It also offers a conceptual framework that can be applied in future studies and encourages further reflection on the psychological mechanisms involved in online interactions between influencers and their audiences.

From a managerial perspective, the results provide valuable insights for companies aiming to optimize their digital communication strategies. A better understanding of consumer perceptions and expectations allows marketers to improve influencer selection, refine promotional messages, and design more effective and authentic marketing campaigns. Moreover, the collected data facilitate improved audience segmentation and the development of personalized marketing actions based on identified behavioral patterns.

Based on the findings, several practical recommendations can be proposed. First, companies should adopt a rigorous selection process when choosing influencers, prioritizing those who demonstrate strong credibility, authenticity, and alignment with brand values rather than merely focusing on popularity. Second, the creation of relevant and engaging content should be emphasized, ensuring that messages are informative, personalized, and consistent with consumer expectations. Third, it is essential to implement performance monitoring systems using specific indicators such as engagement rates, conversion metrics, and brand perception measures to evaluate and optimize influencer campaigns. Finally, marketing teams should be trained in emerging digital influence practices, including legal, ethical, and technological considerations.

Despite its contributions, this study presents certain limitations that open avenues for future research. Future studies could incorporate additional variables such as frequency of exposure to influencer content, level of interaction between influencers and followers, and consumers' emotional engagement. A comparative analysis across different demographic segments (e.g., age, gender, or cultural background) would also provide deeper insights into behavioral differences toward influencer marketing.

Furthermore, qualitative approaches could complement quantitative findings by offering a more nuanced understanding of consumer motivations and resistance to influencer recommendations. It would also be relevant to explore the growing phenomenon of virtual influencers (avatars and AI-based influencers), which raises new ethical, technological, and strategic challenges in digital marketing.

In addition, the evolution of innovative formats such as short-form videos, live shopping, augmented reality, and immersive experiences is progressively redefining the consumer journey and the role of influencers within the conversion funnel. In this context, influencer marketing is no longer merely a promotional tool but has become a strategic lever for co-creating value between brands, influencers, and consumers.

Overall, this research provides a foundational contribution to understanding the role of influencer marketing in shaping consumer purchasing behavior. It highlights the importance of credibility, content relevance, and strategic communication in enhancing purchase intention, while also paving the way for future research aimed at exploring emerging digital marketing practices in an increasingly data-driven and technologically evolving environment.

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Optimization of Machine Tool Downtime Using Predictive Approaches Based on Minimalist Data

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***Abstract*—Minimizing machine tool downtime is crucial in manufacturing due to its impact on production efficiency, profitability, and maintenance costs. Unplanned equipment failures result in production halts, high repair expenses, and prolonged downtime. This study investigates predictive maintenance strategies based on minimalist data (e.g., tool wear, temperature, and vibration) to forecast breakdowns, thereby reducing the reliance on complex and costly monitoring systems. Current AI and ML techniques often capture only correlations rather than true causal relationships, and they struggle with uncertainties in small datasets. To overcome these limitations, our approach employs a probabilistic framework that is rigorously tested to accurately reflect industrial conditions, enhancing both prediction accuracy and risk estimation. The ultimate goal is to develop a cost-effective and reliable solution for small and medium-sized enterprises, improving equipment management, reducing unplanned downtime, and optimizing maintenance planning.**

***Index Terms*—Machine tool downtime, Predictive maintenance, Manufacturing industry, Minimalist data, Artificial Intelligence (AI), Machine Learning (ML), Probabilistic approach**

I. Introduction

In the manufacturing industry, the availability and reliability of machine tools are essential for ensuring efficient and profitable production. Unplanned equipment downtime not only causes significant economic losses but also reduced productivity and high maintenance costs. Predictive maintenance has emerged as an innovative solution to anticipate failures and optimize interventions, thereby minimizing unplanned interruptions.

Traditional maintenance approaches whether preventive or corrective face limitations in terms of cost and effectiveness. Additionally, the advent of Industry 4.0 technologies, particularly Artificial Intelligence (AI) and Machine Learning (ML), has enabled the development of new methods that utilize sensor data to enhance fault diagnosis reliability. However, these solutions often depend on complex and expensive infrastructures, which can hinder their adoption, especially among small and medium-sized enterprises (SMEs).

This study proposes an alternative approach based on the use of minimalist data such as vibrations, temperature, and tool wear to predict equipment failures while reducing the complexity and costs of monitoring systems. By integrating a probabilistic framework with traditional predictive maintenance techniques, our goal is to improve diagnostic accuracy and better assess the risks associated with equipment failures.

The following sections will review the state-of-the-art in predictive maintenance strategies and probabilistic methods in this field, as well as outline the methodology for developing a predictive model tailored to industrial requirements.

II. Concepts and Evolutions

A. Design of Predictive Maintenance

Predictive maintenance relies on the analysis of data collected in real time by sensors integrated into industrial equipment, enabling the anticipation of failures before they occur. Unlike traditional approaches, such as corrective maintenance, which intervenes after a breakdown, or preventive maintenance, scheduled at fixed intervals, this proactive method minimizes unexpected downtime, optimizes resource utilization, and improves profitability.

Advanced sensors measure parameters such as vibrations, temperature, or acoustic emissions, providing valuable indicators to detect anomalies and prevent malfunctions.

B. Technological Evolutions and Advancements

Thanks to the emergence of the Industrial Internet of Things (IIoT) and artificial intelligence (AI), predictive maintenance has reached a new level of performance. AI analyzes massive datasets, identifies complex patterns, and enhances predictions, while tools like Fast Fourier Transform (FFT) enable the early detection of defects such as misalignments or wear. The integration of technologies like infrared thermography and acoustic sensors strengthens the reliability of diagnostics. Finally, self-learning systems based on machine learning offer increased adaptability, solidifying predictive maintenance as a strategic pillar of Industry 4.0.

III. IoT Technologies for Real-Time Monitoring

Predictive maintenance for CNC machines leverages IoT for real-time vibration monitoring. Accelerometers measure vibrations and transmit data for analysis using FFT to detect anomalies and assess machine conditions. A graphical interface compliant with ISO 10816 displays equipment status for informed maintenance decisions. IoT enables predictive maintenance that prevents, consequently monitoring machine conditions via databases like Firebase to prevent unexpected failures. Monitoring four CNC machines showed CNC 2 and 3 in good condition (0.55 and 0.33 mm/s), CNC 1 satisfactory (1.39 mm/s), and CNC 4 requiring intervention (2.08 mm/s). IoT-ML synergy enhances diagnostics, fault detection, and predictive maintenance, reducing downtime and improving efficiency. [1]

IV. Predictive Maintenance and Machine Learning in Industry 4.0

Predictive maintenance in Industry 4.0 integrates IoT, machine learning, and deep learning to anticipate equipment failures. LSTM networks excel in real-time sequential data analysis, while XGBoost efficiently classifies faults. Combining signal processing techniques like wavelet decomposition with deep learning enhances prediction accuracy, reducing downtime and optimizing resources for greater industrial competitiveness. [2]

V. Deep Learning-Based Approach for Estimating the Remaining Useful Life of Machine Tools

Predictive maintenance in CNC milling uses RTF data and AI models to estimate tool wear and RUL, enabling proactive maintenance. Deep learning techniques excel in handling complex data, ensuring accurate predictions and minimizing unexpected failures.

A. Examples of Promising Models

LSTM-Autoencoder networks excel in time-series analysis and anomaly detection through precise signal reconstruction. Hybrid approaches, combining signal processing (FFT, wavelet) with machine learning (Random Forest, Gradient Boosting), enhance detection accuracy under varying conditions.

B. Context and Importance

Predictive maintenance, particularly for CNC milling machines, relies heavily on RTF data to estimate tool wear and predict the RUL. AI-based models have become essential due to their ability to provide accurate predictions and analyze complex data.

C. Process and Innovations

The innovation lies in the use of deep neural networks to model and predict tool wear. Key elements of this process include:

- **Multidomain Feature Extraction:** Relevant features such as entropy and interquartile range (IQR) are extracted from sensor data, which are strongly correlated with tool wear.
- **LSTM-AE Model Training:** The hybrid LSTM-AE model combines an LSTM network to capture temporal dynamics and an Auto-Encoder to model nonlinear relationships between variables.
- **Tool Wear Prediction:** The model is trained on datasets such as PHM10 to predict wear and estimate the RUL.

D. Experiments Conducted and Observations

- **Feature Extraction:** Indicators such as IQR and entropy are identified as strongly correlated with tool wear through Pearson correlation coefficient (PCC) analysis.
- **Construction of the Feature Map:** This map serves as input for the LSTM model to predict target wear.
- **Performance Improvement:** Using wear thresholds and degradation curves allows the generation of accurate RUL estimates, showing significant improvements in MAE and RMSE metrics.

E. Key Results

The model demonstrates exceptional accuracy of 98% in wear prediction, with reduced absolute errors (MAE: $2.6 \pm 0.3222 \times 10^{-3}$; RMSE: $3.1 \pm 0.6146 \times 10^{-3}$). Though the RUL values are slightly underestimated, this approach ensures proactive maintenance planning, thus minimizing unexpected failures.

F. Tool Condition Monitoring (TCM)

The main steps of monitoring include:

1) Fault detection.
2) Identification of fault types.
3) Estimation of the RUL (Remaining Useful Life).

These steps enable the prediction of failures before they occur, ensuring proactive and reliable maintenance.

G. AI-Based Predictive Maintenance Techniques

Approaches integrating AI include:

- **Machine Learning (ML):** Algorithms such as Support Vector Machines (SVM) and Random Forest (RF) are effective in fault classification.
- **Deep Learning (DL):** Advanced deep learning techniques can handle complex scenarios, particularly through sophisticated feature extraction methods.

In specific cases, algorithms like Random Forest have been used to predict tool wear, while XGBoost has been applied to optimize RUL estimation metrics.

VI. Mathematical Formulation for Predictive Maintenance

Either

$$\mathbf{X} = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n,$$

the characteristic vector of a machine tool, and $Y \in \mathbb{R}^+$ the time before a failure. The objective is to estimate the conditional density $f_{Y|X}(y)$ in order to obtain the conditional expectation $E[Y \mid X]$.

A. Conditional Expectation

The mathematical definition is:

$$E[Y \mid X] = \int_0^\infty y\, f_{Y|X}(y)\, dy.$$

For a numerical resolution, we discretize the integral:

$$E[Y \mid X] \approx \sum_{i=1}^{k} y_i\, f_{Y|X}(y_i)\, \Delta y_i,$$

where y_i represents discrete values and Δy the discretization step.

B. Estimate of $f_{Y|X}(y)$

We use a Machine Learning model, noted M, trained on the whole $\{(X(j), y(j))\}_{j=1}^N$ to approximate:

$$M : X \mapsto \{y_1, \dots, y_k\},$$

with

$$p_i \approx \Delta y\, f_{Y|X}(y_i).$$

In the presence of unbalanced data, an oversampling method such as SMOTE is applied to balance the classes.

C. Attention Mechanism via Transformer

To model more finely $f_{Y|X}(y)$, we can use a Transformer model. We first define:

$$\mathbf{Q} = \phi(X) \in \mathbb{R}^d,$$

where ϕ is an encoding function. For each y_i discretized, we associate:

$$\mathbf{K}_i = \phi(y_i), \quad \mathbf{V}_i = \chi(y_i).$$

Or ψ and χ are key and value encoding functions. The attention score is then calculated by:

$$\alpha_i = \frac{\exp\left(\frac{\mathbf{Q}\cdot\mathbf{K}_i}{\sqrt{d}}\right)}{\sum_{j=1}^{k} \exp\left(\frac{\mathbf{Q}\cdot\mathbf{K}_j}{\sqrt{d}}\right)}.$$

The conditional density is modeled by:

$$f_{Y|X}(y_i) \propto \alpha_i,$$

standardized to satisfy:

$$\sum_{i=1}^{k} f_{Y|X}(y_i)\, \Delta y_i = 1.$$

This mathematical approach combines classical probabilistic techniques with advanced machine learning models (Random Forest and Transformers) to estimate the time before a machine fails, based on conditional expectation and the attention mechanism.

VII. Conclusion

This study highlights the potential of predictive maintenance using minimalist data to enhance failure diagnostics and risk assessment while reducing reliance on costly monitoring systems. By integrating traditional predictive techniques with a probabilistic framework, the approach improves accuracy and cost-effectiveness, making it accessible to a wider range of industries.

However, challenges remain in addressing data uncertainty and optimizing model adaptability across different industrial settings. Future work should focus on refining methodologies, conducting comparative analyses, and strengthening empirical validation. Additionally, incorporating high-impact references and ensuring a clearer demonstration of novelty will enhance the study's scientific contribution and practical relevance.

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A penalty approach to solve MPCC problems

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***Abstract*—We consider a new approach that combines penalization and relaxation techniques to solve the MPCC (Mathematical Program with Complementarity Constraints). We prove under the MPCC-Mangasarian-Fromovitz constraints qualifications that any accumulation point of the approximate solution sequence produced by our method is an M-stationary point of the original MPCC. We present some numerical experiments on problems from the MacMPEC library to confirm the efficiency of our approach.**

***Index Terms*—Mathematical programming with complementarity constraints, Penalty function, Regularization techniques.**

I. Introduction

We consider the Mathematical Program with Complementarity Constraints:

$$\begin{aligned} \min_{x\in\mathbb{R}^n} \quad & f(x) \\ \text{s.t.} \quad & h(x)=0,\ g(x)\le 0, \\ & 0\le G(x)\perp H(x)\ge 0, \end{aligned} \qquad \text{(MPCC)}$$

with $f:\mathbb{R}^n\to\mathbb{R}, g:\mathbb{R}^n\to\mathbb{R}^q, h:\mathbb{R}^n\to\mathbb{R}^p$, and $G,H:\mathbb{R}^n\to\mathbb{R}^m$. All these functions are assumed to be continuously differentiable through this paper. The notation $0\le u\perp v\ge 0$ for two vectors u and v in $\mathbb{R}^q$ is a shortcut for $u\ge 0, v\ge 0$ and $u^Tv=0$.

MPCC (Mathematical Programming with Complementarity Constraints) is an important class of problems. It arises frequently in applications in engineering design, economic equilibrium, and multilevel games [23]. MPCC also comes from bilevel programming problems, which have numerous applications in practice [32].

The MPCC is a challenging subclass of non-linear programming problems due to the degeneracy of complementarity constraints. Indeed, the classical Mangasarian-Fromovitz that is very often used to guarantee convergence of algorithms is violated at any MPCC feasible point. This is partly due to the geometry of the complementarity constraint that always has an empty relative interior. These issues have motivated the definition of enhanced constraints qualifications and optimality conditions for (MPCC) as in [10], [11]. In 2005, Flegel and Kanzow [11] defines the "right" necessary optimality condition to (MPCC), it called M(Mordukhovich)-stationary condition. In the literature, there exist a wide variety of MPCC constraints and qualifications, we will focus, in this study, on the MPCC-Mangasarian Fromovitz CQs (MPCC-MFCQ).

A wide range of numerical methods have been proposed to solve this problem such as relaxation methods [8], [9], interior-point methods [21], [25], [30], penalty methods [15], [23], [28], SQP methods [12], dc methods [26], filter methods [20] and Levenberg-Marquardt methods [13]. Among these methods, the relaxation is one of the most popular approaches, which consists of relaxing the complementarity constraints using a parameter, then generates a sequence of non-linear programs, which are more regular than the initial problem. Thus, one can apply the well-studied numerical methods for non-linear programming. Therefore, it is often necessary to prove theoretically that the sequence of approximate solutions converges to a stationary point (or an solution) of the original MPCC.

In [4], [5] the authors have proposed a regularization method which ensures global convergence to stationary point for MPCC with bound constraints. In [18], Kanzow and Schwartz discussed the convergence of the relaxation methods by considering a sequence of approximate stationary points. They proved that these methods may converge to spurious weak-stationary points. In [7], Azizi and Kadrani proposed an approach based on penalty formulation and a relaxation scheme for MPCC, they showed that any accumulation point of the sequence of strong approximate stationary points is a M-stationary point for the MPCC.

We present in this paper a new regularization-penalization method to solve the MPCC : firstly we regularize the complementarity constraints by using concave and nondecreasing functions θ introduced in [14]:

$0\le x\perp y\ge 0,\quad$ is relaxed to $\quad\theta(t,x_i)+\theta(t,y_i)\le 1,$
$i=1,\dots,n.$

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Then we modify the regularization-penalization scheme [16] by using another penalty function. Our objective is to propose a new algorithm that converges to an M-stationary point of the MPCC under the MPCC-Mangasarian-Fromovitz constraints qualifications.

In [16], the authors introduced the parametric barrier $\dfrac{\Delta(z,t)}{1-k\Delta(z,t)}$ to define the penalty function. In our approach, we will consider similar penalty term but we will not need any complicated strategy to update the regularization parameter t since we will consider it as a new variable.

The remainder of the paper is organized as follows. In the next section, we introduce some basic concepts from mathematical programming with complementarity constraints. In Section 3, we present our approximation and formulation. Section 4, is devoted to the convergence proof. In Section 5, numerical experiments are reported on some examples from the MacMpec tests library [19].

Throughout this paper, G_i represents the i-th component of a vector G and similar notations are used for vector-valued functions. ∇f denotes the gradient of a differentiable real value function f defined on $\mathbb{R}^n$. The norm $\|.\|$ denotes the Euclidian norm.

II. Definitions for MPCC and Preliminaries

A. Definitions for MPCC

In this section, we give some definitions and fix some notations for the rest of the paper. Let $\mathcal{Z}$ be the set of feasible points of (MPCC). Given, $x \in \mathcal{Z}$, we denote

$$\begin{aligned}
&\mathcal{I}^{+0}(x) = \{i \in \{1,\dots,q\} \mid G_i(x) > 0 \text{ and } H_i(x) = 0\},\\
&\mathcal{I}^{0+}(x) = \{i \in \{1,\dots,q\} \mid G_i(x) = 0 \text{ and } H_i(x) > 0\},\\
&\mathcal{I}^{00}(x) = \{i \in \{1,\dots,q\} \mid H_i(x) = 0 \text{ and } G_i(x) = 0\},\\
&\mathcal{I}_g(x) = \{i \in \{1,\dots,p\} \mid g_i(x) = 0\}.
\end{aligned}$$

We define the generalized MPCC-Lagrangian function of (MPCC) as

$$\mathcal{L}^r_{MPCC}(x,\lambda) = rf(x) + \lambda^g g(x) + \lambda^h h(x) - \lambda^G G(x)$$
$$-\lambda^H H(x),$$

and denote $\lambda := (\lambda^g, \lambda^h, \lambda^G, \lambda^H)$.

In general, MPCC does not have any KKT stationary point. So we need weaker stationary concepts as in [8], [9].

Definition 2.1 (Stationary point): $x^* \in \mathcal{Z}$ is said

- Weak-stationary if there exist $\lambda \in \mathbb{R}^p \times \mathbb{R}^m \times \mathbb{R}^q \times \mathbb{R}^q$ such that
$$\begin{aligned}
&\nabla_x \mathcal{L}^1_{MPCC}(x^*,\lambda) = 0,\\
&\min(g(x^*),\lambda^g) = 0, h(x^*) = 0,\\
&\forall i \in \mathcal{I}^{+0},\ \lambda^G_i = 0,\ \text{and } \forall i \in \mathcal{I}^{0+},\ \lambda^H_i = 0;
\end{aligned}$$
- Clarke-stationary point if x^* is weak-stationary and
$$\forall i \in \mathcal{I}^{00},\ \lambda^G_i \lambda^H_i \geq 0;$$
- Alternatively (or Abadie)-stationary point if x^* is weak-stationary and
$$\forall i \in \mathcal{I}^{00},\ \lambda^G_i \geq 0 \quad \text{or} \quad \lambda^H_i \geq 0;$$
- Mordukhovich-stationary point if x^* is weak-stationary and
$$\forall i \in \mathcal{I}^{00},\ \text{either} \quad \lambda^G_i > 0, \lambda^H_i > 0 \quad \text{or} \quad \lambda^G_i \lambda^H_i = 0;$$
- Stong-stationary point if x^* is weak-stationary and
$$\forall i \in \mathcal{I}^{00},\ \lambda^G_i \geq 0, \quad \lambda^H_i \geq 0.$$

It is also to be noted that if we assume strict complementarity, i.e. for all $i \in \{1,\dots,q\}$

$$H_i(x) + G_i(x) > 0$$

then all of the stationary conditions presented here are equivalent. For instance, this is the case for the class of binary optimisation problems, whose integer constraints are replaced by complementarity constraints.

There exist a wide variety of MPCC constraint qualification described in the literature. We conclude this section by defining the only one needed later, the MPCC-Mangasarian Fromovitz CQs (MPCC-MFCQ).

Definition 2.2: Let $x^* \in \mathcal{Z}$. MPCC-MFCQ holds at x^* if

$$\sum_{i\in\mathcal{I}_g(x^*)} \alpha_i \nabla g_i(x^*) + \sum_{i=1}^{p} \beta_i \nabla h_i(x^*) + \sum_{i\in\mathcal{I}^{00}(x^*)\cup\mathcal{I}^{0+}(x^*)} \gamma_i \nabla G_i(x^*)$$

$$+ \sum_{i\in\mathcal{I}^{00}(x^*)\cup\mathcal{I}^{+0}(x^*)} \delta_i \nabla H_i(x^*) = 0$$

with $\alpha_i \geq 0, \delta_i \geq 0$ and $\gamma_i \geq 0$. We have $\alpha = \gamma = \delta = 0$.

In view of the constraint qualifications issues that plague the (MPCC), we regularize the complementarity constraints by using some smoothing functions. We present now how we construct this functions.

III. Presentation of γ and θ smoothing functions

We consider a family of smooth functions that are extensively used in various numerical methods. These functions are non-decreasing continuous smooth (C^1) concave functions such that

$\gamma : \mathbb{R} \to]-\infty, 1[$ with $\gamma(x) < 0$ if $x < 0, \gamma(0) = 0,$ and $\lim_{x\to+\infty} \gamma(x) = 1.$

One generic way to build such functions is to consider non-increasing continuous probability density functions $f : \mathbb{R}_+ \to \mathbb{R}_+$ and then take the corresponding cumulative distribution functions

$$\forall x \geq 0, \quad \gamma(x) = \int_0^x f(s)ds.$$

By definition of f

$$\lim_{x\to+\infty} \gamma(x) = \int_0^{+\infty} f(s)ds = 1 \text{ and}$$
$$\gamma(0) = \int_0^0 f(s)ds = 0.$$

The hypothesis on f gives the concavity of θ.

We introduce $\theta(t, x) := \gamma(\frac{x}{t})$ for $t > 0$. This definition is similar to the perspective functions in convex analysis. These functions satisfy $\theta(t, 0) = 0$, and $\forall x > 0, \lim_{t\searrow 0} \theta(t, x) = 1$.
In order to simplify the presentation, we sometimes used the notation $\theta(t, x)$ instead of $\theta_t(x)$. We denote ∂_x or ∂_t the derivative with respect x and t respectively.

From now on, we assume that:

$$\lim_{t\searrow 0} \theta(t, \sqrt{t}) = 1 \quad \text{and} \quad \lim_{t\searrow 0} \partial_x\theta(t, \sqrt{t}) > 0. \qquad (A)$$

Remark 3.1: All these assumptions are not restrictive. Indeed the two functions θ^1 and θ^2 defined below (and all functions in between them) satisfy (A).

$$\forall x \geq 0, \qquad \theta^1(t, x) := \frac{x}{x+t}, \quad \text{and} \quad \theta^2(t, x) := 1-e^{-\frac{x}{t}},$$

and for $x < 0$, $\theta^{1,2}$ is a linear function $\theta^{1,2}(t, x) = ax$ where $a = \partial_x\theta^{1,2}(t, 0) > 0$.
The corresponding derivatives with respect to x are

$$\partial_x\theta^1(t, \sqrt{t}) = \frac{1}{(1+\sqrt{t})^2} \text{ and } \partial_x\theta^2(t, \sqrt{t}) = \frac{1}{t}e^{-\frac{1}{\sqrt{t}}}.$$

A. A θ-smoothing of a complementarity condition

In this paper, we use the θ-functions to regularize the complementarity constraints. The following obvious lemma provides an intuition of the motivation behind such technique.

Lemma 3.2: [4] Given $x, y \in \mathbb{R}_+$, we have

$$x \perp y \Longleftrightarrow \lim_{t\searrow 0} \theta(t, x) + \theta(t, y) \leq 1$$

We prove now some technical lemmas and results that are used in the sequel of the paper to proof the convergence theorem.

Lemma 3.3: $\forall x > 0$, we have

$$\lim_{t\searrow 0} \partial_x\theta(t, x) = 0.$$

We have, $\partial_x\theta(t, x) = \frac{1}{t}\gamma^{'}(\frac{x}{t})$. So,

$$\lim_{t\searrow 0} x.\partial_x\theta(t, x) = \lim_{z\longrightarrow\infty} z\gamma^{'}(z).$$

Let $s > s^{'}$, then by the mean value theorem there exist $c \in [s, s^{'}]$ such that $\gamma(s) - \gamma(s^{'}) = \gamma^{'}(c)(s - s^{'}) \geq 0$.
By the concavity of γ it follows $\gamma(s)-\gamma(s^{'}) \geq \gamma^{'}(s)(s-s^{'})$.
By taking, $s^{'} = \frac{s}{2}$, we have

$$2(\gamma(s) - \gamma(\frac{s}{2})) \geq \gamma^{'}(s)s \geq 0.$$

Using that $\lim_{z\longrightarrow\infty} \gamma(z) = 1$ and passing to the limit, we obtain that

$$\lim_{t\searrow 0} x.\partial_x\theta(t, x) = 0.$$

The result follows since $x > 0$.

Lemma 3.4: Let $t_0 > 0$. For every $t \in (0, t_0]$, we have the two following results:

(i) The set $\{\partial_x\theta(t, x), x \in \mathbb{R}\}$ is bounded, and

(ii) there exist two positive numbers M and M' such that the set $\{\partial_t\theta(t, x), x \geq -t_0\}$ is bounded by $\max\left(\frac{M}{t}, \frac{M^{'}}{t^2}\right)$.

(i) θ is concave with respect x, so the derivative with respect x is decreasing.
On the other hand, θ is increasing so the derivative with respect x is positive. So for $x \geq 0$,

$$0 \leq \partial_x\theta(t, x) \leq \partial_x\theta(t, 0).$$

For $x \leq 0$, we have $\theta_t(x) = \alpha_t x$, with $\alpha_t = \partial_x\theta(t, 0)$. So we have the result.

(ii) We have shown that $\lim_{z\longrightarrow\infty} z\gamma^{'}(z) = 0$. Then, the set $\{z\gamma^{'}(z), z \in [0, +\infty[\}$ is bounded, so there exist $M > 0$ such that $|z\gamma^{'}(z)| < M$, for $z \in [0, +\infty[$.
The derivative with respect t is : $|\partial_t\theta(t, x) = \partial_t\gamma(\frac{x}{t})| = | - \frac{x}{t^2}\gamma^{'}(\frac{x}{t})|$.

We consider two cases: When $x \geq 0, |\partial_t \theta(t,x)| = |-\frac{1}{t}\frac{x}{t}\gamma^{'}(\frac{x}{t})| \leq \frac{M}{t}$.

When $-t_0 \leq x < 0$, $|\partial_t \theta(t,x)| = \gamma^{'}(0)$.
So, for $M' = t_0\gamma^{'}(0)$, we have $|\partial_t \theta(t,x)| = |-\frac{x}{t^2}\gamma^{'}(0)| \leq \frac{M'}{t^2}$.

These functions are further extended as functions from $\mathbb{R}^n$ in $\mathbb{R}^n$ component by component, i.e. for a vector $x \in \mathbb{R}^n$ we have $\theta(t,x) = (\theta(t,x_i))_{1\leq i\leq n}$.

IV. A Penalization-Regularization Approach

The first attempt in the literature to use a relaxation technique for MPCC problems goes back to S. Scholtes in 2001 [31]. In this paper, the author used the following approximation of the complementarity condition

$$\Phi_i^{SS}(G(x),H(x);t) := G_i(x)H_i(x) - t^2; \qquad (SS)$$

It is now, will known that this method converges to a C-stationary point if MPCC-MFCQ holds at its limit point. This relaxation is clearly more regular than (MPCC) since MFCQ is violated at any feasible point of the original problem. The idea to additionally relax the positivity constraints (i.e. consider $\bar{t} > 0$ such that $G(x) \geq -\bar{t}e, H(x) \geq -\bar{t}e$) has been introduced in [27] as an extension to the relaxation (SS).
The relaxation (SS) is very unlikely to satisfy LICQ at a point x^* with $\mathcal{I}^{00}(x^*) \neq \emptyset$, since this would mean that for $t = 0$ three constraints are active for only two gradients.
$G. - H.$ Lin and M. Fukushima proposed in [22] a relaxation with fewer constraints in order to improve the regularity of the relaxed program by considering:

$$\begin{array}{ll} \min\limits_{x\in\mathbb{R}^n} & f(x) \\ \text{s.t.} & h(x) = 0,\ g(x) \geq 0, \\ & G_i(x)H_i(x) \leq t^2, i = 1,\dots,m \\ & (G_i(x)+t)(H_i(x)+t) \geq t^2, i = 1,\dots,m \end{array} \qquad (P^{LF})$$

The authors showed in particular that if MPCC-LICQ holds at a point $x^* \in \mathcal{Z}$, this relaxation satisfies the classical LICQ in any feasible points of P^{LF} in a neighborhood of x^*. Once again, this relaxation converges to a C-stationary point as $t \searrow 0$ as shown in [17].
In [5], Abdallah et al. used the family of smoothing functions introduced in section 3 to approximate the complementarity condition in the simple where $H(x) = x, G(x) = y$ as follows

$$\Phi_i(x,y;t) := \theta(t,x_i) + \theta(t,y_i) - 1, \quad i = 1,\dots,n.$$

with $\Phi(x,y;t) = (\Phi_i(x,y;t))_{1\leq i\leq n}$. In this paper they only considered bound constraints and not general MPCC.

A. Our formulation

In this section, we present our formulation to solve the MPCC problem. We first introduce slack variables u and v to the general nonlinear constraints of the MPCC problem:

$$u = G(x), \quad v = H(x), u \geq 0, v \geq 0.$$

and the slack variables s for the constraints $g(x)$. We obtain

$$\begin{aligned} &\min_z f(x) \\ &\text{s.t. } g(x) - s = 0,\ h(x) = 0, s \geq 0, \\ &\quad H(x) - u = 0, \quad G(x) - v = 0, \\ &\quad 0 \leq u \perp v \geq 0. \end{aligned} \qquad \text{(IV.1)}$$

with $z = (x,u,v,s)$.
This problem can be written as follows

$$\begin{aligned} &\min_z f(x) \\ &\text{s.t. } F(z) = 0, \\ &\quad s \geq 0,\ 0 \leq u \perp v \geq 0. \end{aligned} \qquad (\widetilde{1})$$

where $F(z) := \begin{pmatrix} g(x) - s \\ h(x) \\ H(x) - u \\ G(x) - v \end{pmatrix}$.

This problem has the same properties as the original MPCC problem, in the sense that if x satisfies some MPCC-CQs, and some weak weak or strong stationary conditions for MPCC, the point z satisfies these conditions for $(\widetilde{1})$.

We regularize the complementarity constraints by using continuously, differentiable, non-decreasing and concave θ functions and by the lemma 3.2. We also relax the inequality constraints $v \geq 0$. Consequently, the original MPCC is approximated by the following new smooth parametrized optimization problem:

$$\begin{aligned} &\min_z f(x) \\ &\text{s.t. } F(z) = tw, \\ &\quad \Phi_l(u,v;t) \leq 0, l = 1,\dots,m, \\ &\quad u \geq 0, v \geq -t, s \geq 0. \end{aligned} \qquad (2_t)$$

where $w = (1,\dots,1) \in \mathbb{R}^{p+q+2m}$.

At the limit when the relaxation parameter t is driven to zero, the feasible set of the parametric non-linear program (2_t) converges to the feasible set of the $(\widetilde{1})$. Let $\mathcal{Z}_t$ be the feasible set of (2_t), and $\mathcal{Z}$ the feasible set of $(\widetilde{1})$, it holds that

$$\lim_{t\to 0} \mathcal{Z}_t = \mathcal{Z}.$$

(Every converging sequence of points in $\mathcal{Z}_t$ has its limit in $\mathcal{Z}$)
Now, we consider the penalty function [16] defined as follows

$$f_\sigma(z,t) := \begin{cases} f(x) & \text{if} \quad t = \Delta(z,t) = 0 \\ f(x) + \dfrac{\Delta(z,t)}{2t} + \sigma\beta(t) & \text{if} \quad t > 0, \\ +\infty & \text{otherwise} \end{cases} \tag{IV.2}$$

where $\Delta(z,t) := \|F(z) - tw\|^2$ measures the constraints violation. The function $\beta : [0,\bar{t}] \to [0,\infty)$ is continuously differentiable on $(0,\bar{t}]$ and $\beta(0) = 0$ (for some fixed $\bar{t}$). The term $\sigma\beta(t)$ allows to consider t as a new optimization variable, and minimize simultaneously z and t. In this study, we will take $\beta(t) = \sqrt{t}$.

Our penalized-regularized problem then

$$\begin{aligned} &\min f_\sigma(z,t) \\ &\text{s.t. } \Phi_l(u,v;t) \le 0, l = 1,\dots,m, \\ &\quad u \ge 0, v \ge -t, s \ge 0. \end{aligned} \tag{P_σ}$$

The Lagrangian function of (P_σ) is defined as follows:

$$\begin{aligned} \mathcal{L}_\sigma(z,t,\mu_1,\mu_2,\eta,\nu) := f_\sigma(z,t) - \nu^T s - (\mu_1)^T u \\ -(\mu_2)^T(v + t.\mathbf{1}) + \eta^T\Phi(u,v;t). \end{aligned}$$

Since it is usually not possible to solve P_σ exactly, we will consider a strong ϵ-stationary point of P_σ.

Definition 4.1: We say that $(z,t) \in \mathbb{R}^{n+2m+q} \times \mathbb{R}$ is a strong ϵ-stationary point of P_σ if there exist multipliers ν, μ_1, μ_2, η such that:

$$\|\nabla_z \mathcal{L}_\sigma(z,t)\|_\infty \le \epsilon$$

$$\begin{aligned} &|-\frac{1}{2}t^{-2}\Delta - t^{-1}\sum_{j=1}^{p+q+2m}(F_j - t) \\ &+\sum_{i=1}^{m}(\mu_{2i} + \eta_i(\partial_t\theta(t,u_i) + \partial_t\theta(t,v_i))) + \sigma\frac{1}{2\sqrt{t}}| \le \epsilon \\ &s_i \ge -\epsilon \quad \nu_i \ge 0 \quad |s_i\nu_i| \le \epsilon \\ &u_l \ge -\epsilon \quad \mu_{1,l} \ge 0 \quad |\mu_{1,l}u_l| \le \epsilon \\ &v_l + t \ge -\epsilon \quad \mu_{2,l} \ge 0 \quad |\mu_{2,l}(v_l + t)| \le \epsilon \\ &0 \le \eta \perp \Phi(u,v;t) \le 0 \end{aligned} \tag{IV.3}$$

V. CONVERGENCE ANALYSIS

Before starting our analysis, we need to define different index sets for the active constraints. We will denote for fixed t.

$$\begin{aligned} I_0(z) &= \{i : s_i = 0\}, \\ I_1(z) &= \{l : u_l = 0\}, \\ I_2(z) &= \{l : v_l = -t\}, \\ I_\Phi(z) &= \{l : \Phi_l = 0\}. \end{aligned}$$

and

$$\begin{aligned} I_0^*(z^*) &= \{i : s_i^* = 0\}, \\ I_{0+}^*(z^*) &= \{l : u_l^* = 0, v_l^* > 0\}, \\ I_{+0}^*(z^*) &= \{l : u_l^* > 0, v_l^* = 0\}, \\ I_{00}^*(z^*) &= \{l : u_l^* = 0, v_l^* = 0\}. \end{aligned}$$

We start by two essential lemmas.

Lemma 5.1: For every $t > 0$, we have $I_1(z) \cap I_\Phi(z) = \emptyset$ and $I_2(z) \cap I_\Phi(z) = \emptyset$.
Suppose that $I_1(z) \cap I_\Phi(z) \ne \emptyset$, so there exist an index $i \in I_1(z) \cap I_\Phi(z)$ then $u_i = 0$ and $\Phi_i(u,v) = 0$ so $u_i = 0$ and $\theta(t,u_i) + \theta(t,v_i) - 1 = 0$. This implies that $\theta(t,v_i) = 1$ which impossible for $t > 0$.
We suppose now that that $I_2(z) \cap I_\Phi(z) \ne \emptyset$, so there exist $i \in I_2(z) \cup I_\Phi(z)$ then $v_i = -t$ and $\Phi_i(u,v) = 0$ (i.e. $v_i = -t$ and $\theta(t,u_i) + \theta(t,v_i) - 1 = 0$. This implies that $\theta(t,v_i) > 1$ this impossible for $t > 0$.

Lemma 5.2: Let (z^k,t_k) be a strong ϵ_k-stationary point sequence of the penalized-regularized problem with $\epsilon_k = o(t_k)$. If (z^*,t_*) is a cluster point of the sequence $\{(z^k,t_k)\}$, we have

$$\begin{aligned} &\forall r \in I_\Phi(z^k) \cap (I_{00}^*(z^*) \cup I_{+0}^*(z^*) \cup I_{0+}^*(z^*)), \\ &\lim_{t_k \longrightarrow t_*} \max\left(\partial_{u_r}\theta(t_k,u_r^k), \partial_{v_r}\theta(t_k,v_r^k)\right) > 0. \end{aligned}$$

If $t_* \ne 0$, there is nothing to prove, we only consider the case for $t_* = 0$.
Let $r \in I_\Phi(z^k) \cap I_{0+}^*(z^*)$, in this case, we have $u_r^k \le \sqrt{t_k}$. Indeed, if $u_r^k > \sqrt{t_k}$, so $\theta(t_k,u_r^k) > \theta(t_k,\sqrt{t_k})$ and $\theta(t_k,u_r^k) + \theta(t_k,v_r^k) > \theta(t_k,v_r^k) + \theta(t_k,\sqrt{t_k})$. we obtain $\theta(t_*,u_r^*) + \theta(t_*,v_r^*) > \theta(t_*,v_r^*) + \theta(t_*,\sqrt{t_*})$. This is a Contradiction. On the other hand, the function $\partial_{u_r}\theta(t,.)$ is decreasing since $\theta(t,.)$ is concave, so $\partial_{u_r}\theta(t_k,u_r^k) \ge \partial_{u_r}\theta(t_k,\sqrt{t_k}) > 0$.

For $r \in I_\Phi(z^k) \cap I_{+0}^*(z^*), \partial_{v_r}\theta(t_k,v_r^k) > 0$ (same proof as above).
For $r \in I_\Phi(z^k) \cap I_{00}^*(z^*)$. So, $u_r^k \le \sqrt{t_k}$ or $v_r^k \le \sqrt{t_k}$. Indeed, if $u_r^k > \sqrt{t_k}$ and $v_r^k > \sqrt{t_k}$, so $\theta(t_k,u_r^k) + \theta(t_k,v_r^k) > 2\theta(t_k,\sqrt{t_k})$. For t_k sufficiently small, in the neighborhood of t_* we obtain, $\theta(t_k,u_r^k) + \theta(t_k,v_r^k) > 2$. This is a Contradiction because $r \in I_\Phi(z^k)$. So, as the proof of the previous case, we obtain the result. We study now the relation between the standard qualification constraints of the problem (2_t) and the MPCC. Note that, the reformulation with the slacks variables for the

$(\widetilde{1})$ does not alter the properties of the original MPCC problem. In particular, if we assume that x^* satisfies MPCC-MFCQ for (1), z^* satisfies also MPCC-MFCQ for $(\widetilde{1})$.

Theorem 5.3: Let x^* be a feasible point of the original problem (1) such that MPCC-MFCQ is satisfied at x^*. Then, there exist a neighborhood $\mathcal{U}(z^*)$ of z^* and a sufficiently small scalar $\bar{t} > 0$ such that for any $t \in (0, \bar{t})$, any $z \in \mathcal{U}(z^*) \cap \mathcal{Z}_t$ satisfies the standard MFCQ for the problem (2_t).
satisfies the standard MFCQ for any $z \in \mathcal{U}(z^*) \cap \mathcal{Z}_t$.

Since all the constraints functions of $(\widetilde{1})$ are continuously differentiable (the constraints functions of the MPCC problem are continuously differentiable) then, there exist a neighborhood $\mathcal{U}_1(z^*)$ and a positive scalar $\bar{t}_1$ such that $\forall t \in (0, \bar{t}_1)$ and for every $z \in \mathcal{U}_1(z^*) \bigcap \mathcal{Z}_t$ we have:

$$\begin{aligned} I_0(z) &\subseteq I_0^*(z^*) \\ I_1(z) &\subseteq I_{00}^*(z^*) \cup I_{0+}^*(z^*) \\ I_2(z) &\subseteq I_{00}^*(z^*) \cup I_{+0}^*(z^*) \end{aligned}$$

Since $(\widetilde{1})$-MFCQ is satisfied at z^* for the problem $(\widetilde{1})$, the vectors $\{\nabla F_j | j = 1, \dots p+q+2m\} \cup \{-e_{n+i} | i \in I_0^*(z^*)\} \cup \{e_{n+q+k} | k \in I_{00}^*(z^*) \cup I_{0+}^*(z^*)\} \cup \{e_{n+q+m+l} | l \in I_{00}^*(z^*) \cup I_{+0}^*(z^*)\}$ are positively linearly independent where e_l is the unit vector of size $n+q+2m$.

We have:

$$\begin{aligned} & I_1(z) \cup (I_\Phi(z) \cap I_{0+}^*(z^*)) \cup (I_\Phi(z) \cap I_{00}^*(z^*)) \\ & \qquad \subseteq I_{00}^*(z^*) \cup I_{0+}^*(z^*) \\ & I_2(z) \cup (I_\Phi(z) \cap I_{+0}^*(z^*)) \cup (I_\Phi(z) \cap I_{00}^*(z^*)) \\ & \qquad \subseteq I_{00}^*(z^*) \cup I_{+0}^*(z^*) \end{aligned} \tag{V.1}$$

By the proposition 2.2 [29] there exist a neighborhood $\mathcal{U}_2(z^*)$ and a positive constant $\bar{t}_2$ sufficiently small such that for all points $z \in \mathcal{U}_2(z^*) \bigcap \mathcal{Z}_t$ with $t \in (0, \bar{t}_2)$, the following vectors

$$\begin{aligned} & \{\nabla F_j | j = 1, \dots p+q+2m\} \cup \{-e_{n+i} | i \in I_0(z)\} \\ & \cup \{e_{n+q+k} | k \in I_1(z)\} \cup \{e_{n+q+m+l} | l \in I_2(z)\} \\ & \cup \{e_{n+q+r} | r \in I_\Phi(z) \cap I_{0+}^*(z^*)\} \\ & \cup \{e_{n+q+r} | r \in I_\Phi(z) \cap I_{00}^*(z^*)\} \\ & \cup \{e_{n+q+m+k} | k \in I_\Phi(z) \cap I_{+0}^*(z^*)\} \\ & \cup \{e_{n+q+m+k} | k \in I_\Phi(z) \cap I_{00}^*(z^*)\} \end{aligned} \tag{V.2}$$

are positively linearly independent.

Inspired by Lemma 5.2, we can multiply any terms by some positive number and the new family vectors will remains positively linearly independent.

$$\begin{aligned} & \{\nabla F_j | j = 1, \dots p+q+2m\} \cup \{-e_{n+i} | i \in I_0(z)\} \\ & \cup \{e_{n+q+k} | k \in I_1(z)\} \cup \{e_{n+q+m+l} | l \in I_2(z)\} \\ & \cup \{\partial_{u_r}\theta(t_k, u_r) e_{n+q+r} | r \in I_\Phi(z) \cap I_{0+}^*(z^*)\} \\ & \cup \{\max(\partial_{u_r}\theta(t, u_r), \partial_{v_r}\theta(t, v_r))\, e_{n+q+r} | r \in I_\Phi(z) \cap I_{00}^*(z^*)\} \\ & \cup \{\partial_{v_r}\theta(t, v_r) e_{n+q+r} | r \in I_\Phi(z) \cap I_{+0}^*(z^*)\} \\ & \cup \{\partial_{v_k}\theta(t, v_k) e_{n+q+m+k} | k \in I_\Phi(z) \cap I_{+0}^*(z^*)\} \\ & \cup \{\max(\partial_{u_r}\theta(t, u_r), \partial_{v_r}\theta(t, v_r))\, e_{n+q+m+k} | k \in I_\Phi(z) \cap I_{00}^*(z^*)\} \\ & \cup \{\partial_{u_k}\theta(t, u_k) e_{n+q+m+k} | k \in I_\Phi(z) \cap I_{0+}^*(z^*)\} \end{aligned} \tag{V.3}$$

are positively linearly independent.

Now we have to check that MFCQ holds for the problem (2_t) for $z \in \mathcal{U} \cap \mathcal{Z}_t$, with $\mathcal{U} = \mathcal{U}_1 \cap \mathcal{U}_2$ and $\bar{t} = \min\{\bar{t}_1, \bar{t}_2\}$. We should to show that :

$$\begin{aligned} & \sum_{j=1}^{p+q+2m} \lambda_j \nabla F_j - \sum_{i \in I_0(z)} \nu_i e_{n+i} + \sum_{k \in I_1(z)} \mu_{1,k} e_{n+q+k} \\ & + \sum_{l \in I_2(z)} \mu_{2,l} e_{n+q+m+l} + \sum_{r \in I_\Phi(z)} \eta_r^\Phi \partial_{u_r}\theta(t, u_r) e_{n+q+r} \\ & + \textstyle\sum_{r \in I_\Phi(z)} \eta_r^\Phi \partial_{v_r}\theta(t, v_r) e_{n+q+m+r} = 0. \end{aligned} \tag{V.4}$$

with $\lambda_j \geq 0, \nu_i \geq 0, \mu_{1,k} \geq 0, \mu_{2,l} \geq 0, \eta_r \geq 0$, holds only for the trivial solution ($\lambda_j = \nu_i = \mu_{1,k} = \mu_{2,l} = \eta_r^\Phi = 0$).

By spliting the index set I_Φ: $I_\Phi(z) = (I_\Phi(z) \cap I_{+0}^*(z^*)) \cup (I_\Phi(z) \cap I_{00}^*(z^*)) \cup (I_\Phi(z) \cap I_{0+}^*(z^*))$. From (V.3) it follows that:

$$\begin{cases} \lambda_j = 0, \nu_i = 0, \mu_{1,k} = 0, \mu_{2,l} = 0, \\ \eta_r^\Phi \partial_{u_r}\theta(t, u_r) = 0 \quad \text{for } r \in I_\Phi(z) \cap I_{0+}^*(z^*) \\ \eta_r^\Phi \max(\partial_{u_r}\theta(t, u_r), \partial_{v_r}\theta(t, v_r)) = 0 \quad \text{for } r \in I_\Phi(z) \cap I_{00}^*(z^*) \\ \eta_r^\Phi \partial_{v_r}\theta(t, v_r) = 0 \quad \text{for } r \in I_\Phi(z) \cap I_{+0}^*(z^*) \end{cases}$$

By lemma 5.2, we obtain that $\eta_r = 0$ so the standard MFCQ are satisfied for the problem (2_t) .

The following theorem identifies a relation between the optimal solutions of the original MPCC problem and the penalized-regularized problem under the MPCC-MFCQ assumption.

Denote, for k be sufficiently large $I_S := \{i/u_i^* > 0, v_i^* > 0\}$.

Theorem 5.4: Let $\{\sigma_k\}$ be a non-decreasing sequence converging to $+\infty$. Let (z^k, t_k) be a strong ϵ_k-stationary point of the penalized-regularized problem with $\epsilon_k = o(t_k)$. If (z^*, t_*) is a cluster point of $\{(z^k, t_k)\}$, z^* is feasible for $(\widetilde{1})$ and the MPCC-MFCQ holds at z^*, then:

i) $t_* = 0$, and
ii) If $I_S \cap supp(\eta^*) = \emptyset$ then z^* is an M-stationary point.
If $I_S \cap supp(\eta^*) \neq \emptyset$ then z^* is an C-stationary point.

i) The first line of the definition of the strong $\epsilon-$ stationary is as follows:

$$\|\nabla_x f^k + t_k^{-1} \sum_{j=1}^{p+q+2m} (F_j^k - t_k w) \nabla_x F_j^k\|_\infty \leq \epsilon_k$$

$$\begin{array}{c} \big| -t_k^{-1}(F_i^k - t_k w_i) - \nu_i^k \big| \leq \epsilon_k, \quad i = 1, \dots, q \\ \big| -t_k^{-1}(F_{p+q+l}^k - t_k w_l) - \hat{\mu}_{1,l}^k \big| \leq \epsilon_k, \quad l = 1, \dots, m \\ \big| -t_k^{-1}(F_{p+q+m+l}^k - t_k w_l) - \hat{\mu}_{2,l}^k \big| \leq \epsilon_k, \quad l = 1, \dots, m \end{array} \tag{V.5}$$

with

$$\begin{array}{rcl} F_i^k & = & F_i(z^k) \\ \widetilde{\mu}_{1,l}^k & = & \mu_{1,l}^k - \eta_l^k \partial_{u_l} \theta(t_k, u_l^k) \\ \widetilde{\mu}_{2,l}^k & = & \mu_{2,l}^k - \eta_l^k \partial_{v_l} \theta(t_k, v_l^k) \end{array}$$

We will prove by contradiction that if $t_* \neq 0, \sigma_k$ can not converge to $+\infty$. We suppose there exist a subsequence (z^k, t_k) that converges the $\{(z^*, t_*)\}$ such that $t_* \neq 0$. Let $\pi^k = -t_k^{-1}(F(z^k) - t_k w)$, this sequence is bounded because F_j are continuously differentiable, so the sequence $\{(\nu_i^k, \widetilde{\mu}_{1,l}^k, \widetilde{\mu}_{2,l}^k)\}$ is bounded.

By the second line of the definition of the strong $\epsilon-$ stationary (Definition.3)

$$\big| -\tfrac{1}{2} t_k^{-2} \Delta_k - t_k^{-1} \sum_{j=1}^{p+q+2m} (F_j^k - t_k w) + \sum_{i=1}^{m} (\mu_{2,i}^k + \eta_i^k (\partial_t \theta(t_k, u_i^k) + \partial_t \theta(t_k, v_i^k))) + \sigma_k \frac{1}{2\sqrt{t_k}} \big| \leq \epsilon_k.$$

By multiplying by $\sqrt{t_k}$ we obtain

$$\big| -\tfrac{1}{2} t_k^{-3/2} \Delta_k - t_k^{-1/2} \sum_{j=1}^{p+q+2m} (F_j^k - t_k w) + t_k^{1/2} \sum_{i=1}^{m} (\mu_{2,i}^k + \eta_i^k (\partial_t \theta(t_k, u_i^k) + \partial_t \theta(t_k, v_i^k))) + \frac{\sigma_k}{2} \big| \leq \epsilon_k \sqrt{t_k} \tag{V.6}$$

In this inequality the first and second terms of the left side are both bounded.
We have to show that the third term is bounded. This term can be written as

$$\sum_{i=1}^{m} \left(\mu_{2,i}^k + \eta_i^k (\partial_t \theta(t_k, u_i^k) + \partial_t \theta(t_k, v_i^k))\right) = \sum_{i \in I_\Phi(z^k)} \left(\mu_{2,i}^k + \eta_i^k (\partial_t \theta(t_k, u_i^k) + \partial_t \theta(t_k, v_i^k))\right) + \sum_{i \notin I_\Phi(z^k)} \left(\mu_{2,i}^k + \eta_i^k (\partial_t \theta(t_k, u_i^k) + \partial_t \theta(t_k, v_i^k))\right)$$

We have to prove that these two sums are bounded :

- For $l \notin I_\Phi(z^k)$ then $\eta_l^k = 0$ (by the definition of the strong $\epsilon-$ stationary). We have $\widetilde{\mu}_{2,l}^k = \mu_{2,l}^k - \eta_l^k \partial_{v_r} \theta(t_k, v_r^k)$, so $\widetilde{\mu}_{2,l}^k = \mu_{2,l}^k$ and since $\widetilde{\mu}_{2,l}^k$ is bounded then $\mu_{2,l}^k$ is bounded .

- For $l \in I_\Phi(z^k)$ then $\theta(t_k, u_l^k) + \theta(t_k, v_l^k) - 1 = 0$. By lemma 5.1, we have $I_1(z^k) \cap I_\Phi(z^k) = \emptyset$ and $I_2(z^k) \cap I_\Phi(z^k) = \emptyset$, so, $l \notin I_1(z^k)(u_l^k > 0)$ and $l \notin I_2(z^k)(v_l^k > -t_k)$.
By the definition of the strong $\epsilon-$ stationary), we have :
$$-\epsilon_k \leq \mu_{2,l}^k (v_l^k + t_k) \leq \epsilon_k$$
since t_k tends to $t_* \neq 0$, there exists $b > 0$ such that $t_k \geq b$ for all k.
$$0 \leq \mu_{2,l}^k \leq \frac{\epsilon_k}{t_k} \leq \frac{\epsilon_k}{b}.$$
Then, $\mu_{2,l}^k$ is bounded.
Now, let us to show that η_l^k is bounded.
We have
$$\widetilde{\mu}_{2,l}^k = \mu_{2,l}^k - \eta_l^k \partial_{v_l} \theta(t_k, v_l^k),$$
then $|\eta_l^k| = \frac{|\mu_{2,l}^k - \widetilde{\mu}_{2,l}^k|}{\partial_{v_l} \theta(t_k, v_l^k)}$. $\widetilde{\mu}_{2,l}^k$ and $\mu_{2,l}^k$ are bounded, it remains to show that $\partial_{v_l} \theta(t_k, v_l^k)$ is bounded below. Since the sequence $v_l^k \longrightarrow v_l^*, v_l^k$ is bounded and there exist $\lambda > 0$ such that $v_l^k < \lambda$. In other hand, $t_k \longrightarrow t_* \neq 0$ then there exist $b > 0$ such that $t_k > b \neq 0$. By the concavity of θ by respect to the second variable:
$$\partial_{v_l} \theta(t_k, v_l^k) \geq \partial_{v_l} \theta(t_k, \lambda) \geq \partial_{v_l} \theta(b, \lambda) > 0.$$

So, $\partial_{v_l}\theta(t_k, v_l^k)$ is bounded below and we have the result.

Since $\sigma_k \to \infty$, we obtain a contradiction in (V.6), so, we can conclude that $t_* = 0$.

ii)We can write (V.5) as:

$$\begin{array}{l}\|\nabla_x f^k + \sum_{i=1}^{q}[(\nu_i^k - \Pi_i^k) - \nu_i^k]\nabla_x F_i^k \\ + \sum_{l=1}^{m}[(\hat{\mu}_{1,l}^k - \Pi_{p+q+l}^k) - \hat{\mu}_{1,l}^k]\nabla_x F_{p+q+l}^k \\ + \sum_{l=1}^{m}[(\hat{\mu}_{2,l}^k - \Pi_{p+q+m+l}^k) - \hat{\mu}_{2,l}^k]\nabla_x F_{p+q+m+l}^k \\ - \sum_{j=1}^{p}\Pi_{q+j}^k)\nabla_x F_{p+j}^k\|_\infty \le \epsilon_k \\ \left|\nu_i^k - \Pi_i^k\right| \le \epsilon_k, \quad i = 1, \dots, q \\ \left|\hat{\mu}_{1,l}^k - \Pi_{p+q+l}^k - \right| \le \epsilon_k, \quad l = 1, \dots, m \\ \left|\hat{\mu}_{2,l}^k - \Pi_{p+q+m+l}^k\right| \le \epsilon_k, \quad l = 1, \dots, m \end{array} \tag{V.7}$$

Let's show that $\{(\nu^k, \beta^k, \hat{\mu}_1^k, \hat{\mu}_2^k)\}$ is bounded, where β^k denotes the subsequence restricted to coefficients of equality constraints.
Assume that the sequence $\{(\nu^k, \beta^k, \hat{\mu}_1^k, \hat{\mu}_2^k)\}$ is unbounded, then we can find a subsequence such that

$$\frac{(\nu^k, \beta^k, \hat{\mu}_1^k, \hat{\mu}_2^k)}{\| (\nu^k, \beta^k, \hat{\mu}_1^k, \hat{\mu}_2^k) \|} \longrightarrow (\overline{\nu}, \overline{\beta}, \overline{\mu}_1, \overline{\mu}_2) \neq 0$$

Using the inequalities (V.7), and since the gradient vectors of F are bounded (F is continuously differentiable), and dividing by the term $\| (\nu^k, \beta^k, \hat{\mu}_1^k, \hat{\mu}_2^k) \|$, we obtain at the limit:

$$0 = \sum_{i=1}^{q}\overline{\nu}_i\nabla_x F_i^* + \sum_{j=1}^{p}\overline{\beta}_j\nabla_x F_{j+q}^* + \sum_{l=1}^{m}\overline{\mu}_{1,l}\nabla_x F_{p+q+l}^* + \sum_{l=1}^{m}\overline{\mu}_{2,l}\nabla_x F_{p+q+m+l}^*$$

with $\nabla_x F_i^* = \nabla_x F_i(z^*)$.
Now, we have to show that $\overline{\nu} \ge 0, \overline{\nu}_i = 0, i \notin I_0^*(z^*)$, $\overline{\mu}_{1,l} = 0, l \notin I_{00}^*(z^*) \cup I_{0+}^*(z^*)$ and $\overline{\mu}_{1,l} = 0, l \notin I_{00}^*(z^*) \cup I_{+0}^*(z^*)$.

The definition of the strong $\epsilon-$ stationary we have $\overline{\nu} \ge 0$. Suppose that $\overline{\nu}_i^k > 0$, so there exist some constant $c > 0$ and all k sufficiently large. This yields $|s_i^k| \le \frac{\epsilon_k}{c} \longrightarrow 0$, so $s_i^* = 0$.
Let $l \in \{1, \dots, m\}$ such that $u_l^* > 0$ and suppose that $\overline{\mu}_{1,l} \neq 0$. We have $u_l^k \longmapsto u_l^*$ and $|\hat{\mu}_{1,l}^k| > c$ for k sufficiently large. By the definition of the strong $\epsilon-$ stationary we have $0 \le \mu_{1,l}^k \le \frac{\epsilon_k}{u_l^k} \longrightarrow 0$.
But, $\widetilde{\mu}_{1,l}^k = \mu_{1,l}^k - \eta_l^k\partial_{u_l}\theta(t_k, u_l^k) \longrightarrow 0$ (for $u_l^* > 0$ we have $\partial_{u_l}\theta(t_k, u_l^k) \to 0$), so we obtain a contradiction. We deduce that $\overline{\mu}_{1,l} = 0$, same proof for $\overline{\mu}_{2,l}$.

So, we have the following positive linear combination:

$$\begin{array}{l}\sum_{i\in I_0^*(z^*)}\overline{\nu}_i\nabla_z F_i^* + \sum_{j=1}^{p}\overline{\theta}_j\nabla_z F_{j+q}^* \\ + \sum_{I_{00}^*(z^*)\cup I_{0+}^*(z^*)}\overline{\mu}_{1,l}\nabla_z F_{p+q+l}^* \\ + \sum_{I_{00}^*(z^*)\cup I_{+0}^*(z^*)}\overline{\mu}_{2,l}\nabla_z F_{p+q+m+l}^* + \sum_{i\in I_0^*(z^*)}\overline{\nu}_i e_{p+i} \\ + \sum_{k\in I_{00}^*(z^*)\cup I_{0+}^*(z^*)}\overline{\mu}_{1,l}e_{p+q+k} \\ + \sum_{l\in I_{00}^*(z^*)\cup I_{+0}^*(z^*)}\overline{\mu}_{2,l}e_{p+q+m+l} = 0\end{array}$$

By the given that MPCC-MFCQ is satisfied at z^* and the previous theorem, implies $(\overline{\nu}_i, \overline{\theta}_j, \overline{\mu}_{1,l}, \overline{\mu}_{2,l}) = 0$, contradiction.
Consequently the sequence $(\nu^k, \theta^k, \hat{\mu}_1^k, \hat{\mu}_2^k)$ is bounded and so there exist a subsequence that converges to some limit point $(\nu^*, \theta^*, \hat{\mu}_1^*, \hat{\mu}_2^*)$, which satisfies the following conditions:

$$\begin{array}{l}\nabla_x f^* - (\nu^*, \theta^*, \hat{\mu}_1^*, \hat{\mu}_2^*)^T\nabla_x F^* = 0 \\ \nu^* \ge 0, supp(\nu^*) \subseteq I_0^*, \\ supp(\mu_1^*) \subseteq I_{00}^*(z^*) \cup I_{0+}^*(z^*), \\ supp(\mu_2^*) \subseteq I_{00}^*(z^*) \cup I_{+0}^*(z^*).\end{array}$$

So, z^* is W-stationary point.

Suppose now that there exist some indexes l such that $u_l^* = v_l^* = 0$. By taking a sub-sequence of (z^k, t_k) strong $\epsilon-$ stationary point, in these situations, we can only meet the following situations
Case 1: For every k assigned $u_l^k = 0, v_l^k \ge 0 > -t_k$.
By the definition of the strong $\epsilon-$ stationary

$$0 \le \mu_{2,l}^k(v_l^k + t_k) \le \epsilon_k$$

then

$$0 \le \mu_{2,l}^k \le \frac{\epsilon_k}{v_l^k + t_k} \le \frac{\epsilon_k}{t_k}$$

when $k \to \infty$, $\mu_{2,l}^k \to 0$ since $\epsilon_k = o(t_k)$.
On the other hand, we have $\theta(t_k, u_l^k) + \theta(t_k, v_l^k) - 1 \neq 0$, then $\eta_l^k = 0$.
$\widetilde{\mu}_{2,l}^k = \mu_{2,l}^k - \eta_l^k\partial_{v_l}\theta(t, v_l^k) \to \widetilde{\mu}_{2,l}^* = 0$.

Case 2: $u_l^k = 0, (v_l^k \ge -t_k$ and $v_l^k < 0)$.
In this case, we have $\theta(t_k, u_l^k) + \theta(t_k, v_l^k) - 1 \neq 0$, so

$\eta_l^k = 0$.
Then, $\widetilde{\mu}_{1,l}^k = \mu_{1,l}^k \geq 0$ and $\widetilde{\mu}_{2,l}^k = \mu_{2,l}^k \geq 0$ so $\widetilde{\mu}_{1,l}^* \geq 0, \widetilde{\mu}_{2,l}^* \geq 0$.

Case 3: $u_l^k > 0, v_l^k < 0, v_l^k \geq -t_k$.
Same proof as case 2.

Case 4: $u_l^k > 0, v_l^k \geq 0$, and $(\theta(t_k, u_l^k) + \theta(t_k, v_l^k) - 1 \neq 0)$:
Same proof as case 1.
Case 5: $u_l^k > 0, v_l^k > 0$ and $(\theta(t_k, u_l^k) + \theta(t_k, v_l^k) - 1 = 0)$. By the definition of the strong $\epsilon-$ stationary

$$0 \leq \mu_{1,l}^k u_l^k \leq \epsilon_k$$

then

$$\mu_{1,l}^k \leq \frac{\epsilon_k}{t_k}$$

when $k \to \infty$, $\mu_{1,l}^k \to 0$ since $\epsilon_k = o(t_k)$.
Same, by the definition of the strong $\epsilon-$ stationary,

$$0 \leq \mu_{2,l}^k \leq \frac{\epsilon_k}{v_l^k + t_k} \leq \frac{\epsilon_k}{t_k}$$

when $k \to \infty$, $\mu_{2,l}^k \to 0$ since $\epsilon_k = o(t_k)$.
So, $\widetilde{\mu}_{1,l}^* \widetilde{\mu}_{2,l}^* = (\eta_l^*)^2 \partial_{u_l} \theta(t_k, u_l^*) \partial_{v_l} \theta(t_k, u_l^*)$.
If $I_S \cap \eta_l^* \neq \emptyset$, we can meet the case 5 and we obtain a C-stationary point.
If $I_S \cap \eta_l^* = \emptyset$ then we obtain a M-stationary point.

VI. Numerical results

In this section, we present some preliminary numerical results. These simulations have been done using AMPL language [6], with the the KNITRO solver. Our aim is just to verify the qualitative numerical efficiency of our approach. Our algorithm to solve the original MPCC is :

Algorithm

1. Choose a starting point (z^0, t_0) and σ_0 . Set $k = 0$.

2. While the stopping criterion for the MPCCs is not satisfied (i.e. $\Delta(z^k; t_k) > tol$) do
(Δ is defined in (IV.2)).
Find an approximate solution (z^{k+1}, t_{k+1}) of the penalized-relaxed problem (P_σ) , by using (z^k, t_k) as a starting point.
Let $\sigma_{k+1} = \alpha\sigma_k$.

We use a subset of the MacMPEC [19] test problems with known optimal values and solutions. In all our tests, we use the same function β defined by $\beta(t) := \sqrt{t}$ as in [16]. The starting point is almost the default value given by the solver and does not influence the solution. The obtained solution is obtained when standard approximate stationary and the feasibility conditions are satisfied. The feasibility is reached if the constraints violation measure and the variable t are equal to zero up to the prescribed tolerance.
The following tables give for each considered problem and for different starting points, the final value of the variable t_*, the objective value (Obj.val) of the MPCC, the constraint violation Δ_*, and the final value of the penalty parameter σ_*. In our experiments, we used $tol = 1e-5$ and we made a logarithmic scaling for our two functions to bound their gradients.
Each constraint

$$\theta(u_i, t) + \theta(v_i, t) \leq 1$$

in the case of the θ_t^1 function

$$1 - \frac{t}{u_i + t} + 1 - \frac{t}{v_i + t} \geq 1$$

is in fact replaced by the following inequality

$$\ln\left(\frac{t}{u_i + t} + \frac{t}{v_i + t}\right) \geq 0,$$

in the case of the θ_t^1 function and

$$t \ln\left(e^{-\frac{u_i}{t}} + e^{-\frac{v_i}{t}}\right) \geq 0.$$

and in the case of the θ_t^2 function.

TABLE I
USING θ^1-FUNCTION

Problem	t_*	Start	Obj.val	Δ^*	σ_*
Bard1A	$1.17e-05$	n/a	16.9999	$2.56e-09$	8
bilevel1	$1e-09$	n/a	-0.00063	$9.88e-08$	16
bilevel2	$5e-11$	n/a	-6600	$1.8e-11$	32768
bilevel3	$1e-06$	n/a	12.6787	$1.09e-11$	2
dempe	$1e-07$	n/a	28.25	$4.90e-13$	16
desilva	$1e-07$	n/a	-0.99999	$3.10e-12$	2
df1	$1e-09$	n/a	$1.87e-13$	$3.29e-22$	2
Gauvin	$1.4e-05$	n/a	20	$6.21e-9$	4
hs044-i	$4e-06$	(1,1,1,1)	$7.46e-06$	$1e-19$	2
jr1	$1e-08$	n/a	0.5	$1e-16$	2
scholtes1	$1e-07$	n/a	2.0	$9.57e-14$	2
nash1	$7e-05$	nash1a.data	$1.7e-08$	$2.05e-09$	2
	0.0002	nash1b.data	$7.35e-06$	$1.14e-08$	2
	0.0001	nash1c.data	$3.99e-06$	$1.61e-09$	2
	$4e-06$	nash1d.data	$3.13e-06$	$1.56e-10$	2
	0.0001	nash1e.data	$7.3e-07$	$3.73e-09$	2
gnash1	$2e-07$	gnash10.data	-230.82	$2.54e-11$	32768
	$1e-05$	gnash11.data	-129.91	$3.33e-08$	256
	$1e-06$	gnash12.data	-36.93	$6.1e-07$	32768
stackelberg1	$1e-10$	n/a	-3266.67	$9.58e-15$	512

VII. Conclusion

We proposed a penalization and a relaxation scheme to solve the MPCC problems. Under the MPCC-Mangasarian-Fromovitz constraints qualifications, we showed the link between the penalized problem and the MPCC, by proving that any accumulation point of the sequence of strong approximate first-order points

TABLE II
USING θ^2-FUNCTION

Problem	t_*	Start	Obj.val	Δ^*	σ_*
Bard1A	$1e-08$	n/a	17	$6e-13$	2
bilevel1	$1.2e-06$	n/a	$1.7e-05$	$5.17e-10$	4
bilevel2	$1e-10$	n/a	-6600	$1.37e-12$	32768
bilevel3	$1e-06$	n/a	-7.32144	$5.5e-07$	$9e+15$
dempe	$1e-07$	n/a	28.25	$4.90e-13$	16
desilva	$2e-06$	n/a	-0.9999	$3.9e-12$	2
df1	$1e-09$	n/a	$1.2e-09$	$1.2e-18$	2
Gauvin	$2.68e-08$	n/a	20	$2e-13$	4
jr1	$1e-08$	n/a	0.5	$1e-16$	2
scholtes1	$1e-09$	n/a	2.25	$2.9e-18$	2
nash1	$1e-07$	nash1a.data	$5.4e-11$	$2.5e-11$	2
	$4e-08$	nash1b.data	$3.4e-11$	$1e-12$	2
	$2.6e-07$	nash1c.data	0.5	$1e-11$	2
	$4.3e-08$	nash1d.data	0.5	$6.6e-12$	2
	$5e-08$	nash1e.data	$4.6e-09$	$5.8e-12$	2
hs044-i	$1.7e-05$	$(1,1,1,1)$	$7.2e-06$	$2.25e-09$	2
gnash1	$6.5e-08$	gnash10.data	-230.823	$2e-07$	32768
	$1e-05$	gnash11.data	-129.915	$2.9e-08$	256
	$1e-06$	gnash12.data	-36.9331	$1.8e-10$	64
stackelberg1	$1e-10$	n/a	-3266.67	$5.48e-15$	256

generated by our scheme is M-stationary for MPCC. To illustrate the effectiveness of our approach, we tested our method on several examples from the MacMpec library and obtained very promising results; This a first step and we plan in future work, to consider realistic and large-scale problems.

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