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Numerical Simulation of Transient Heat Conduction in Cylindrical Geometry Using an Axisymmetric Crank–Nicolson Scheme

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***Abstract*—Heat transfer in cylindrical geometries is central to a wide range of industrial processes, from tubular heat exchangers in power generation to metallurgical treatment of cylindrical components. While analytical solutions of the heat equation in cylindrical coordinates offer valuable insights, they are typically restricted to idealized cases with simplified boundary conditions. To overcome these limitations, this work presents a robust Python-based solver employing the Crank Nicolson finite-difference scheme in axisymmetric coordinates. The framework accommodates Dirichlet, Neumann, and Robin boundary conditions, accounts for axial gradients, and allows initial radial perturbations, providing flexibility f or r ealistic s cenarios. Numerical stability is ensured through implicit integration, with a Rannacher start-up phase effectively suppressing spurious oscillations. The solver is validated through transient heat conduction simulations in iron, steel, and platinum cylinders, highlighting the role of thermal diffusivity in shaping temperature distributions and diffusion dynamics. Results show that high-diffusivity materials, such as platinum, achieve faster thermal homogenization, whereas steel demonstrates greater thermal inertia. Computational performance analysis further underscores the trade-offs between dense and sparse implementations, with potential optimizations via LU factorization reuse and iterative solvers. Beyond delivering physical insights, the developed tool serves as a pedagogical platform for studying conduction mechanisms in cylindrical systems.**

[2]*Index Terms*—Heat transfer, Cylindrical geometry, Crank Nicolson scheme, Implicit methods, Python

I. Introduction

Heat transfer in cylindrical geometry is a central issue in many industrial processes. Indeed, cylindrical configurations are found in a wide range of applications, from tubular heat exchangers widely used in power plants and the chemical industry to metallurgical processes such as ingot solidification, rolling, or heat treatment of cylindrical parts. In these contexts, controlling heat fluxes i s e ssential t o e nsure system performance, durability, and safety. For example, in a heat exchanger, overall efficiency directly depends on the ability of cylindrical tubes to transfer heat between two fluids at different temperatures. Similarly, in metallurgical processes, precise control of thermal gradients helps avoid residual stresses, cracking, or undesirable deformations. However, cylindrical geometry imposes specific m athematical c onstraints, particularly related to the axisymmetric nature of the problem and the singularity of the radial term near the central axis, which makes modeling the phenomenon both rich and complex. Thus, the study of heat transfer in cylindrical geometry is of great importance, both theoretically and practically, for understanding and optimizing high-value industrial processes [1]–[3].

Given this importance, analytical solutions of the heat equation in cylindrical coordinates have long been a key reference. These solutions, often derived using series expansions or integral transforms, provide an exact description of the phenomenon but only in highly idealized situations [4]. They generally assume simple boundary conditions, such as imposed temperature (Dirichlet), zero or constant flux (Neumann), or uniform convection (Robin) on well-defined surfaces. Moreover, they mainly apply to infinite or semi-infinite geometries, or to steady-state regimes without large spatial variations. In real industrial settings, however, initial and boundary conditions are rarely so simple: they can be heterogeneous, time-dependent, or involve multiple physical mechanisms simultaneously. For instance, cooling a metallic cylinder with an external fluid may involve non-uniform convection along its height, while the inner surface is maintained at a variable temperature. In such cases, analytical solutions become impractical, as they cannot incorporate all the geometric and physical particularities of the problem. Consequently, although analytical approaches are valuable for validating certain reference cases, they are not sufficient by themselves to meet the modeling requirements of real-world applications [5], [6].

In this context, numerical methods have gained increasing importance, as they overcome the limitations of analytical approaches by offering great flexibility. Among them, implicit methods stand out as particularly robust and efficient tools for solving the transient heat equation. Unlike explicit schemes, which are subject to restrictive stability constraints (CFL condition), implicit methods allow the use of larger time steps without risk of numerical instability. This results in considerable computational efficiency, especially when simulating long-duration processes [7], [8]. The Crank Nicolson scheme, in particular, represents an ideal compromise between accuracy and stability: it is second-order in time, improving result quality, while remaining unconditionally stable. In cylindrical geometry, its use is especially advantageous as it properly handles radial and axial derivatives, including near the axis of symmetry where singularities may arise. Variants such as

the Rannacher start-up further enhance the method's reliability by avoiding numerical oscillations caused by overly abrupt initial conditions. In short, implicit numerical methods are an indispensable approach for addressing realistic cylindrical thermal problems, combining rigor, efficiency, and stability [9].

With this perspective, the present contribution proposes the development of a flexible Python code dedicated to simulating heat transfer in cylindrical geometry, based on the Crank Nicolson scheme in axisymmetric coordinates. The code accounts for different boundary conditions, whether walls are maintained at constant temperature (Dirichlet), subject to imposed fluxes (Neumann), or exchanging heat convectively with an external fluid (Robin). Axial-end temperatures can be independently fixed, thus enabling the modeling of imposed thermal gradients along the cylinder's height. The initial condition can also be enriched with the addition of a radial perturbation, simulating situations where thermal heterogeneities exist from the initial instant. By using standard libraries such as NumPy for linear algebra and Matplotlib for visualization, the tool is both efficient and accessible. Beyond its academic value, this code serves as a pedagogical resource, allowing exploration of various thermal conduction scenarios and assessment of the impact of physical, geometric, and numerical parameters on the cylinder's thermal behavior. In doing so, it concretely illustrates the contribution of implicit numerical methods to solving heat transfer problems in cylindrical geometry and paves the way for possible extensions to even more complex industrial cases.

II. Related Work

A. Bibliometric Analysis

To provide an overview of the research landscape on transient heat conduction in cylindrical geometry using implicit numerical schemes, a bibliometric analysis was conducted. The search string applied was:

> ("transient heat conduction" OR "unsteady heat conduction" OR "heat diffusion") AND ("cylindrical geometry" OR "cylinder" OR "axisymmetric") AND ("Crank-Nicolson" OR "implicit scheme" OR "finite difference" OR "numerical simulation")

The bibliometric dataset covers the period **1990 2024**, with a total of **77 documents** retrieved from **60 different sources**. The average annual growth rate is **3.28%**, and the average number of citations per document is **18.08**, indicating that the field has a moderate but steady academic impact.

1) Evolution of Publications Over Time: Figure 1 presents the temporal distribution of publications. The trend reveals an irregular growth pattern, with sporadic peaks in publication numbers, such as in **1993, 2012, 2015, and 2020**. The year 2020 marks the highest peak with **7 documents**, reflecting a growing interest in the application of numerical schemes to heat transfer problems during that period. Despite fluctuations, the overall trajectory shows an increasing research activity over the last three decades, confirming the sustained relevance of the topic.

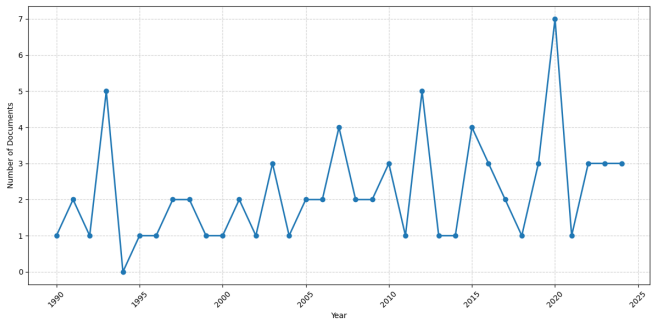

Fig. 1. Evolution of documents on transient heat conduction in cylindrical geometry (1990 2024).

2) Distribution by Document Type: Table I reports the distribution of publication types. The majority of contributions are **journal articles (87%)**, demonstrating the preference of researchers for peer-reviewed dissemination. **Conference papers (10.4%)** also play a notable role, while only a small fraction of the output consists of **book chapters and conference reviews (1.3% each)**. This distribution underlines the academic maturity of the field, where rigorous articles dominate while conferences provide a platform for early-stage discussions.

TABLE I
Distribution of document types.

Document type	Count	Percentage
Article	67	87.0%
Conference Paper	8	10.4%
Book Chapter	1	1.3%
Conference Review	1	1.3%
Total	77	100%

3) Geographical Distribution of Research: Figure 2 highlights the contributions by country. The **United States** dominates the field with **24 documents**, followed by **China (10 documents)**. Other active contributors include **Germany, Iran, Japan, Brazil, Canada, and Morocco**, each with between 4 and 5 publications. Emerging contributions are observed from **Chile and India**. This distribution suggests that the research community is highly internationalized, reflecting the global significance of transient heat conduction in industrial and scientific applications.

4) Keyword Analysis: In addition to temporal and geographical trends, keyword analysis was performed to identify the main thematic areas of research. Figure 3 presents the word cloud generated from the author keywords and Keywords Plus.

The most frequent terms include **heat conduction**, **finite difference method**, **cylinders**, **numerical methods**, and **boundary conditions**, which align with the mathematical and computational modeling focus of the field. Keywords such as **thermal conductivity**, **temperature distribution**, and **heat transfer** highlight the physical quantities of primary interest. Emerging terms like **axisymmetric**, **nonlinear equations**, and **computer simulation** reflect the increasing complexity

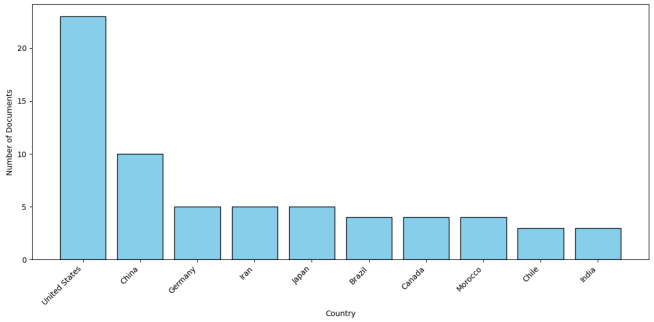

Fig. 2. Top contributing countries in the field.

of studied models and the reliance on advanced numerical approaches.

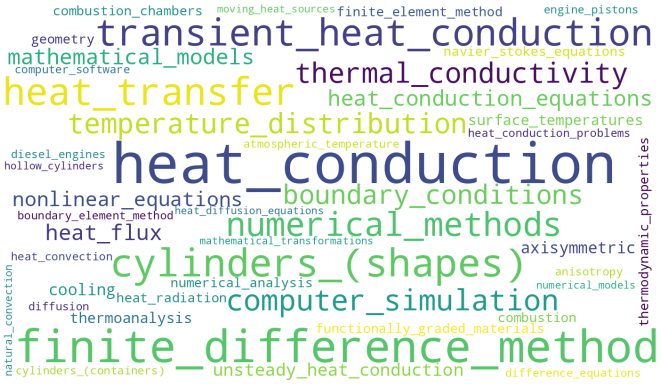

Fig. 3. Word cloud of the most frequent keywords in the dataset.

5) Author Collaboration and Productivity: According to the bibliometric metadata, **204 authors** contributed to the retrieved corpus, with only **5 single-authored documents**, confirming a strong trend toward collaborative research. On average, each document involves **2.96 co-authors**, which reflects the interdisciplinary nature of the field, requiring expertise in heat transfer, numerical analysis, and computational methods.

B. Qualitative Analysis

The study of transient heat conduction in cylindrical and related geometries has been the subject of extensive research, with different approaches developed to address the challenges of stability, accuracy, and complex boundary conditions. Early works established the limitations of analytical solutions, which are often restricted to simple geometries and boundary conditions. For instance, the analytical resolution of transient conduction in composite cylindrical domains with internal heat sources provides exact benchmarks but remains limited in scope and applicability to realistic problems [10]. Such solutions are valuable for validation but cannot handle heterogeneous materials or time-varying boundary conditions.

To overcome these limitations, numerical methods have been widely applied. One of the most influential studies compared various time-marching schemes for transient heat conduction, highlighting the issue of numerical "noise" associated with the Crank Nicolson scheme [11]. This work demonstrated that while Crank Nicolson is unconditionally stable, it may suffer from oscillations unless corrective techniques such as averaging or modified start-up steps are applied. These insights remain highly relevant for the current study, where a Rannacher start-up strategy is employed to mitigate oscillations.

The finite element method (FEM) has also been widely used to solve transient heat conduction problems, particularly in applications involving complex geometries. For example, FEM was applied to power cables to capture transient temperature distributions under dynamic electrical loading [12]. The study showed that FEM offers flexibility in handling irregular boundaries, material discontinuities, and convective losses situations where finite difference approaches struggle. Similarly, FEM formulations have been developed for buried cables under transient thermal loads, confirming its suitability for engineering applications requiring detailed spatial resolution [14]. These contributions illustrate the importance of robust spatial discretization techniques in practical engineering systems.

In parallel, finite-difference approaches have been refined to improve stability and accuracy. A study on nonlinear transient heat conduction in wood demonstrated the ability of implicit finite-difference schemes to capture strongly nonlinear behavior due to variable thermal properties [13]. This highlighted the importance of accounting for material heterogeneity, which is essential for realistic industrial applications. Another contribution investigated transient conduction from two parallel isothermal cylinders using bipolar cylindrical coordinates [15]. The study combined Fourier series expansion with a Crank Nicolson discretization to handle the complex interaction between neighboring cylinders, illustrating the versatility of hybrid analytical-numerical methods for multi-body configurations.

More recently, numerical studies have extended these ideas to general axisymmetric and cylindrical geometries. For example, finite-difference simulations of transient conduction in composite cylindrical media explored the effects of internal heat generation and material discontinuities [16]. The results emphasized that diffusivity contrasts strongly affect thermal gradients, a conclusion consistent with experimental observations in metallurgical processes. Similarly, analytical and semi-analytical techniques for cylindrical conduction problems remain an active research field, as seen in studies focusing on layered or eccentric cylinders, although their practical application is often limited by simplifying assumptions [10], [15].

Overall, the literature demonstrates a clear evolution from purely analytical treatments to robust numerical schemes capable of handling realistic geometries and material complexities. Crank Nicolson remains a reference method thanks to its accuracy and stability, though adaptations are often needed to eliminate oscillations [11]. Finite element and finite difference methods have been successfully applied to engineering problems such as power cables [12], [14], wood conduction

[13], and multi-cylinder systems [15]. The present work builds on these foundations by implementing an axisymmetric Crank Nicolson scheme in cylindrical coordinates, combining stability with flexibility in boundary condition treatment. Unlike earlier works focused on specific applications, this study emphasizes a generalizable computational framework that can be adapted to a variety of industrial scenarios.

III. Methodology

A. Physical Model

The transient heat conduction in a homogeneous solid cylinder is governed by the axisymmetric heat equation:

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right), \tag{1}$$

where $T(r, z, t)$ is the temperature, and $\alpha = \frac{k}{\rho c_p}$ is the thermal diffusivity, with k the thermal conductivity, ρ the density, and c_p the specific heat capacity. The computational domain is a cylinder of radius R and height L. Axisymmetry eliminates dependence on the azimuthal coordinate θ, reducing the problem to two spatial dimensions (r, z).

B. Boundary Conditions

The following boundary conditions are considered:

- **Axial boundaries:**

$$T(r, 0, t) = T_{\text{bottom}}, \quad T(r, L, t) = T_{\text{top}}. \tag{2}$$

- **Radial boundary at $r = R$:**
  - Dirichlet: $T(R, z, t) = T_{\text{wall}}$,
  - Neumann: $\frac{\partial T}{\partial r}(R, z, t) = 0$,
  - Robin (convection):

$$-k \frac{\partial T}{\partial r}(R, z, t) = h \left(T(R, z, t) - T_\infty(z, t) \right). \tag{3}$$

- **Axis of symmetry at $r = 0$:**

$$\frac{\partial T}{\partial r}(0, z, t) = 0. \tag{4}$$

C. Initial Condition

The initial temperature distribution is prescribed as a linear interpolation between axial boundaries:

$$T(r, z, 0) = T_{\text{bottom}} + \frac{z}{L} \left(T_{\text{top}} - T_{\text{bottom}} \right). \tag{5}$$

To investigate radial diffusion, a perturbation (radial bump) can be added:

$$T(r, z, 0) \leftarrow T(r, z, 0) + A \exp \left(- \left(\frac{r}{r_0} \right)^2 \right), \tag{6}$$

where A is the amplitude and r_0 a characteristic radial length.

D. Numerical Method

1) Spatial Discretization: The spatial domain is discretized using a structured finite-difference grid with $(N_r+1)\times(N_z+1)$ nodes, step sizes $\Delta r = R/N_r$ and $\Delta z = L/N_z$.

For an interior node (i, j), the cylindrical Laplacian is approximated by:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \Big|_{i,j} \approx \frac{T_{i-1,j} - 2T_{i,j} + T_{i+1,j}}{\Delta r^2} + \frac{1}{2 r_i \Delta r} \left(T_{i+1,j} - T_{i-1,j} \right). \tag{7}$$

with $r_i = i\Delta r$.

The axial derivative is discretized as:

$$\frac{\partial^2 T}{\partial z^2} \Big|_{i,j} \approx \frac{T_{i,j-1} - 2T_{i,j} + T_{i,j+1}}{\Delta z^2}. \tag{8}$$

At the axis $r = 0$, the singularity is removed using the symmetry condition:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \Big|_{0,j} \approx \frac{4}{\Delta r^2} \left(T_{1,j} - T_{0,j} \right). \tag{9}$$

2) Time Discretization: Time integration is carried out using the θ-method:

$$\frac{T^{n+1} - T^n}{\Delta t} = \alpha \left[\theta L T^{n+1} + (1 - \theta) L T^n \right], \tag{10}$$

which gives the linear system:

$$A T^{n+1} = B T^n + f^{n+1}, \tag{11}$$

with

$$A = I - \theta \alpha \Delta t L, \quad B = I + (1 - \theta) \alpha \Delta t L, \tag{12}$$

and f^{n+1} includes Robin boundary contributions.

The scheme is unconditionally stable for $\theta \geq 0.5$. A Rannacher start-up (few steps with $\theta = 1$) is employed to damp initial oscillations, before switching to Crank–Nicolson ($\theta = 0.5$) for accuracy.

3) Boundary Condition Implementation:

- Dirichlet: rows in A and B replaced by identity.
- Neumann: included directly in L stencil.
- Robin: coefficients of A and right-hand side vector modified accordingly.

E. Post-processing

The solution vector is reshaped into a 2D field $T(r, z)$ at each time step. Post-processing includes:

- 2D maps of $T(r, z)$
- radial and axial temperature profiles

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IV. Results and Discussion

In this section, we analyze the numerical results obtained for three representative materials: iron, steel, and platinum. The temperature distributions are computed using the axisymmetric Crank–Nicolson scheme after $t = 1300$ s of transient evolution. The bottom and top boundaries are imposed at $T = 15°\text{C}$ and $T = 23°\text{C}$, respectively, while the lateral wall exchanges heat with an external fluid at $T_\infty = 18°\text{C}$ through a Robin-type convective boundary condition.

A. Thermophysical properties of materials

Table II summarizes the thermal properties used for each material. The thermal diffusivity $\alpha = k/(\rho c_p)$, which controls the rate of heat propagation, varies significantly between materials and directly influences the observed temperature fields.

TABLE II
Thermophysical properties of the studied materials.

Material	k [W/mK]	ρ [kg/m^3]	c_p [J/kgK]	α [m^2/s]
Iron	80	7860	452	2.25×10^{-5}
Steel	50	7950	490	1.29×10^{-5}
Platinum	70	21450	130	2.51×10^{-5}

B. Iron results

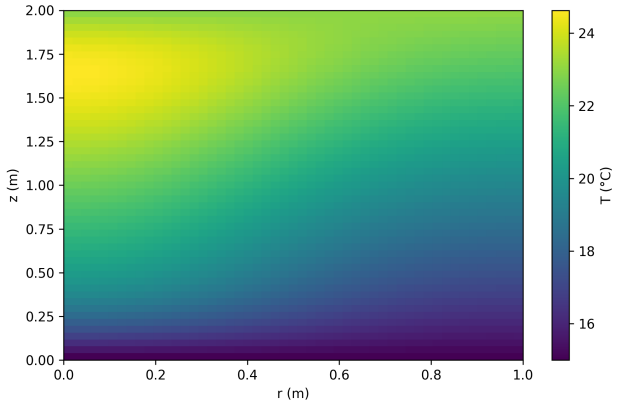

Fig. 4. Temperature distribution in the iron cylinder ($k = 80$ W/mK, $\rho = 7860$ kg/m^3, $c_p = 452$ J/kgK).

Figure 4 shows the final temperature field for iron. The axial gradient dominates the distribution, reflecting the boundary conditions imposed at the top and bottom surfaces. Due to the relatively high diffusivity of iron ($\alpha \approx 2.25 \times 10^{-5}$ m^2/s), heat spreads efficiently across the domain, leading to smooth isotherms and limited radial gradients.

C. Steel results

Figure 5 illustrates the case of steel. With the lowest diffusivity of the three materials ($\alpha \approx 1.29 \times 10^{-5}$ m^2/s), heat propagates more slowly, resulting in sharper gradients near the bottom boundary. Thermal inertia is more pronounced in this case, delaying homogenization of the temperature field. This characteristic can be advantageous in industrial processes where damping of thermal fluctuations is required.

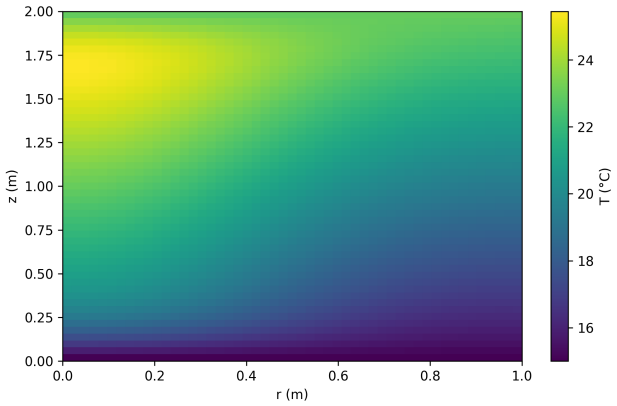

Fig. 5. Temperature distribution in the steel cylinder ($k = 50$ W/mK, $\rho = 7950$ kg/m^3, $c_p = 490$ J/kgK).

D. Platinum results

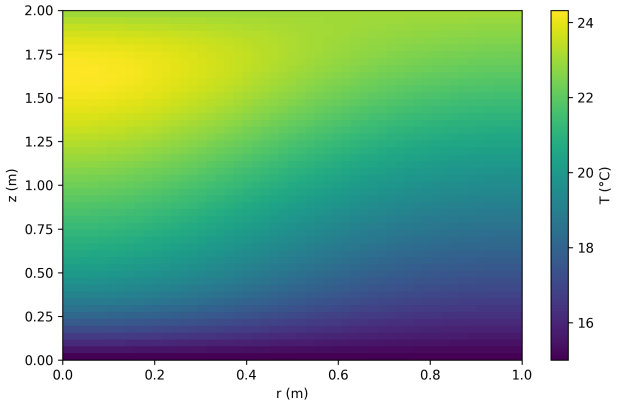

Fig. 6. Temperature distribution in the platinum cylinder ($k = 70$ W/mK, $\rho = 21450$ kg/m^3, $c_p = 130$ J/kgK).

Figure 6 presents the results for platinum. Despite its intermediate conductivity, the very high density combined with the low specific heat capacity yields the largest diffusivity among the three materials ($\alpha \approx 2.51 \times 10^{-5}$ m^2/s). Consequently, heat diffuses more rapidly, leading to smoother axial profiles and reduced radial gradients. This property makes platinum particularly suitable for applications requiring rapid thermal response, such as sensors or catalytic components.

E. Comparative discussion

The comparison of the three materials highlights the critical role of thermal diffusivity in transient conduction:

- **Iron:** intermediate behavior, with relatively fast homogenization due to moderate diffusivity.
- **Steel:** lowest diffusivity, resulting in stronger gradients and higher thermal inertia.

- **Platinum:** highest diffusivity, promoting rapid propagation and uniformization of temperature.

These results demonstrate that material selection directly governs the characteristic diffusion time. From an industrial perspective, steel is suitable for applications requiring thermal damping, whereas iron and platinum are more favorable when rapid heat transfer and homogenization are desired. The results confirm the ability of the Crank–Nicolson scheme to capture the essential physics of transient conduction in cylindrical geometries.

F. Computational complexity of the solver

Let N_r and N_z denote the numbers of radial and axial intervals (code variables: `Nr` and `Nz`). The total number of nodes is

$$n \equiv n_{\text{nodes}} = (N_r + 1)(N_z + 1).$$

With time step Δt (code: `dt`) and final time t_{final} (code: `t_final`), the number of time steps is

$$N_{\text{steps}} = \left\lceil \frac{t_{\text{final}}}{\Delta t} \right\rceil \quad \text{(code: } \texttt{n\_steps}\text{)}.$$

a) Assembly.: The discrete operator L comes from a 5-point stencil in (r, z), so the arithmetic to fill coefficients is $O(n)$. In the current code, however, L, A, and B are stored as dense $n \times n$ arrays (`numpy ndarray`), leading to $O(n^2)$ memory. Forming

$$A = I - \theta\alpha\Delta t L, \qquad B = I + (1-\theta)\alpha\Delta t L$$

also costs $O(n^2)$ in time. With sparse storage, both time and memory would be $O(n)$.

b) Per time step (current dense implementation).: At each step, one dense matrix–vector product BT^n is computed ($O(n^2)$), followed by solving

$$AT^{n+1} = \text{RHS}.$$

The call `numpy.linalg.solve` performs a full LU factorization at each step, which is $O(n^3)$ in time and $O(n^2)$ in memory; the triangular solves are $O(n^2)$. Thus, as written:

$$\text{Time} = O(N_{\text{steps}}\, n^3), \qquad \text{Memory} = O(n^2).$$

c) Reusing the factorization within phases.: Within each phase, A is constant: first in the Rannacher start-up with $\theta = 1$ (`build_AB(1.0)`), then in the Crank–Nicolson phase with $\theta = \frac{1}{2}$ (`build_AB(0.5)`). If one factorizes A only once per phase (e.g., with `lu_factor`/`lu_solve`) and reuses it, the complexity becomes:

$$\text{Preprocessing per phase: } O(n^3), \qquad \text{Per step: } O(n^2).$$

Hence, the total cost is

$$O\big(n^3 + N_{\text{steps}}\, n^2\big).$$

d) Sparse and iterative improvements.: Each row of L has only $O(1)$ nonzero entries due to the stencil. Using sparse storage reduces memory to $O(n)$ and matrix–vector products to $O(n)$. Sparse direct solvers (depending on fill-in) or preconditioned iterative solvers (CG, GMRES) can further reduce runtime while preserving the accuracy of the Crank–Nicolson scheme.

e) Summary.:

- Current dense implementation (`solve` at each step): $O(N_{\text{steps}}\, n^3)$ time, $O(n^2)$ memory.
- With LU reuse per phase: $O(n^3 + N_{\text{steps}}\, n^2)$.
- With sparse solvers: $O(n)$ memory and $O(n)$ per iteration, with total cost depending on fill-in (direct) or iteration count (iterative).

V. Conclusion and Perspectives

The present study addressed the modeling and simulation of transient heat conduction in cylindrical geometries through the development of a Crank Nicolson based Python solver. Starting from the governing axisymmetric heat equation, the implementation incorporated robust treatments of spatial discretization, time integration, and diverse boundary conditions, including Dirichlet, Neumann, and Robin formulations. The methodology was carefully designed to handle the singularity at the cylinder axis while ensuring stability and accuracy through implicit time-stepping. The adoption of a Rannacher start-up further enhanced numerical reliability, preventing oscillations often encountered in conventional Crank Nicolson schemes.

Application of the solver to three representative materials iron, steel, and platinum illustrated the influence of thermophysical properties on transient conduction behavior. Platinum, with the highest thermal diffusivity, exhibited rapid temperature homogenization, confirming its suitability for applications requiring quick thermal response. Iron demonstrated intermediate behavior, while steel, with the lowest diffusivity, retained stronger gradients and thermal inertia, making it preferable where damping of thermal fluctuations is beneficial. These comparative analyses highlight the critical role of material selection in industrial heat transfer processes, underscoring the practical value of computational modeling.

From a computational perspective, the solver was shown to scale unfavorably in its dense-matrix form, with per-step complexity of $O(n^3)$. However, substantial improvements are attainable by reusing LU factorizations within time-integration phases, reducing the effective cost to $O(n^3 + N_{\text{steps}} n^2)$. Further efficiency gains can be realized by adopting sparse storage and solvers, which reduce memory to $O(n)$ and enable iterative approaches that exploit the sparsity of the finite-difference operator. This dual perspective capturing physical insights while analyzing computational performance provides a solid foundation for extending the code to larger and more complex domains.

The study contributes both academically and pedagogically. On the academic side, it demonstrates how implicit finite-difference schemes can overcome the limitations of analytical solutions, offering flexibility in handling complex geometries

and heterogeneous boundary conditions. Pedagogically, the implementation in Python, relying on widely available scientific libraries, makes the solver accessible to students and practitioners alike. It serves as a didactic tool for exploring the effects of geometry, boundary conditions, and material properties on conduction processes.

Future directions include extending the solver to account for nonlinear material properties, phase-change phenomena, or coupling with convective and radiative modes. Integration with sparse iterative solvers and adaptive meshing strategies would further enhance computational efficiency, paving the way for tackling large-scale industrial applications. Overall, the developed framework successfully balances rigor, efficiency, and flexibility, advancing the numerical study of heat conduction in cylindrical systems.

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